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AN EXTENDED KALMAN FILTER FIRE CONTROL SYSTEM AGAINST AIR-TO-AI--ETC(U)
DEC 77 S J CUSUMANO, M DE PONTE
AFIT/GE/EE/77-13-VOL-2
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AN EXTENDED KALMAN FILTER FIRE CONTROL

SYSTEM AGAINST AIR-TO-AIR MISSILES (II)

THESIS

AFIT/GE/EF/77-13 Salvatore J. Cusumano Capt USAF and Manuel De Ponte, Jr. Capt USAF

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(14) AFIT/GE/EE/77-13-VOL-2

AN EXTENDED KALMAN FILTER FIRE CONTROL SYSTEM
AGAINST AIR-TO-AIR MISSILES.

VOLUME II.

THESIS

9 Master's thesis,

Presented to the Faculty of the School of Engineering

of the Air Force Institute of Technology

Air University

in Partial Fulfillment of the

Requirements for the Degree of

Master of Science

by

Salvatore J./Cusumano

Manuel/De Ponte, Jr

Graduate Engineering

December 1977

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### Volume II

# Table of Contents

Page	е
Appendix B1	
Introductionl	
Organization2	
Zero Order Missile Filter4	
Third Order Missile Filter15	
Benchmark29	
Fourth Order Missile Filter (A/P at 0 sec)41	
Fourth Order Missile Filter (A/P at 3 sec)53	
Fourth Order Missile Filter (A/P at 5 sec)65	
Fourth Order Missile Filter (complete linearization)77	
Fourth Order Missile Filter (using R-K integration).89	
Sensitivity Analysis (n = 6.)101	
Sensitivity Analysis (n = 3.)	
Sensitivity Analysis $(\tau_2 = .8)$	
Sensitivity Analysis $(\tau_2 = .1)$	
Sensitivity Analysis (M = 2.)149	
Sensitivity Analysis (M = 8.)	
First Order Filter ( $\tau_f$ set equal to .85)	
n Estimation - n Initialized at 3	
n Estimation - n Initialized at 6200	
Tf Estimation - Tf Initialized at 1.5	
$\tau_{\mathbf{f}}$ Estimation - $\tau_{\mathbf{f}}$ Initialized at .3	
M/S Estimation - M/S Initialized at 45244	
M/S Estimation - M/S Initialized at 15260	

# Table of Contents (Cont'd)

									Pa	ige
n	and	$\tau_{\mathbf{f}}$	Estima	ation	(high-	g sce	enario).		2	76
n	and	$\tau_{\mathbf{f}}$	Estima	ation	(low-8	gscer	nario)	• • • • • •	2	92
n,	$\tau_{\mathbf{f'}}$	ar	nd M/S	Estin	ation.				3	808
n,	$\tau_{f}$	an	nd M/S	Estin	nation	with	Initial	State	Errors.3	128

#### Appendix B

#### Introduction

This Appendix contains the graphical results of the Monte Carlo analysis of this study. The plots will be presented in sets. All sets will include the dynamic state error plots for:

 $V_{mx}^{\mathbf{I}}$  - x-velocity for the missile in the inertial frame

 $\boldsymbol{\theta_{_{\boldsymbol{T}}}}$  - Line-of-sight angle as seen by the ownship

R - Range

 $\dot{R}$  - Range rate (relative closing velocity where, by convention,  $\dot{R}$  = - $V_c$ 

 $\mathbf{a}_{\mathbf{L}}$  - Missile's developed lateral acceleration. And when estimated, the parameter error plots for:

n - Proportional navigation constant

 $\tau_{\mathbf{f}}$  - Time constant of the first order lag

M/S - Ratio of mass to cross sectional area will be included in the set. In addition to the error plots, a tuning plot (covariance matching) will be included for each state and parameter. The smooth curve on the tuning plot represents the square root of the filter-calculated covariance. The "apparently noisy" curve represents the square root of the variance of the error calculated over the twenty simulations. Also included, in various sets, are covariance convergence plots (described in Chapter IV). The box in the upper right hand corner of each plot indicates whether the plot is a mean error plot or a covariance plot. The order of the missile as well as the data set number are also in-

cluded. The data set number indicates the tuning parameters used for a particular run. The filter initial estimates, the tuning parameters and a brief description of the purpose of each set will be included at the beginning of each set. It should be noted that the high-g scenario was used for all cases except the one annotated low-g.

#### Organization

The sets will be arranged in the following order:

Figure	Subject
1-9	Zero order missile filter
10-21	Third order missile filter
22-31	Benchmark
32-41	Fourth order missile filter (A/P at 0 sec)
42-51	Fourth order missile filter (A/P at 3 sec)
52-61	Fourth order missile filter (A/P at 5 sec)
62-71	Fourth order missile filter (A/P at 0 sec -
	complete linearization of $\underline{f}$ )
72-81	Fourth order missile filter (A/P at 5 sec -
	using fourth order Runge-Kutta integration
	package)
82-91	Sensitivity analysis (n = 6.)
92-101	Sensitivity analysis (n = 3.)
102-111	Sensitivity analysis ( $\tau_2 = .8$ )
112-121	Sensitivity analysis $(\tau_2 = .1)$
122-131	Sensitivity analysis $(M = 2.)$
132-141	Sensitivity analysis (M = 8.)
142-151	First order missile filter ( $\tau_{f}$ set equal to .85)

152-164	n estimation - n initialized at 3.
165-178	n estimation - n initialized at 6.
179-190	$\tau_{\mathbf{f}}$ estimation - $\tau_{\mathbf{f}}$ initialized at 1.5
191-202	$\tau_{\mathbf{f}}$ estimation - $\tau_{\mathbf{f}}$ initialized at .3
203-216	M/S estimation - M/S initialized at 45.
217-230	M/S estimation - M/S initialized at 15.
231-244	n and $\tau_{\mathbf{f}}$ estimation (high-g scenario)
245-258	n and $\tau_f$ estimation (low-g scenario)
259-276	n, $\tau_f$ , and M/S estimation
277-294	n, $\tau_f$ , and M/S estimation with the dynamic
	states initialized with some error.

### Zero Order Missile Filter

The initial state estimates and the tuning parameters for this case are

$$V_{mx}^{I}(0) = 1225.7 \text{ fps}$$
 $\theta_{I}(0) = 4.363345 \text{ radians}$ 
 $R(0) = 10000. \text{ feet}$ 
 $\dot{R}(0) = -2122. \text{ fps}$ 
 $a_{IJ}(0) = 0.$ 
 $n(0) = 4.5$ 
 $\tau_{f}(0) = N/A$ 
 $M/S(0) = 29.197 \text{ slugs/ft}^{2}$ 

$$\underline{\mathbf{R}} = \begin{bmatrix} 4.E-5 & 0. & 0. \\ 0. & 500. & 0. \\ 0. & 0. & 100. \end{bmatrix}$$

These plots were generated to demonstrate the performance of this filter with the software changes noted in Appendix A. The parameters n and M/S were not estimated, but were set to their correct values.

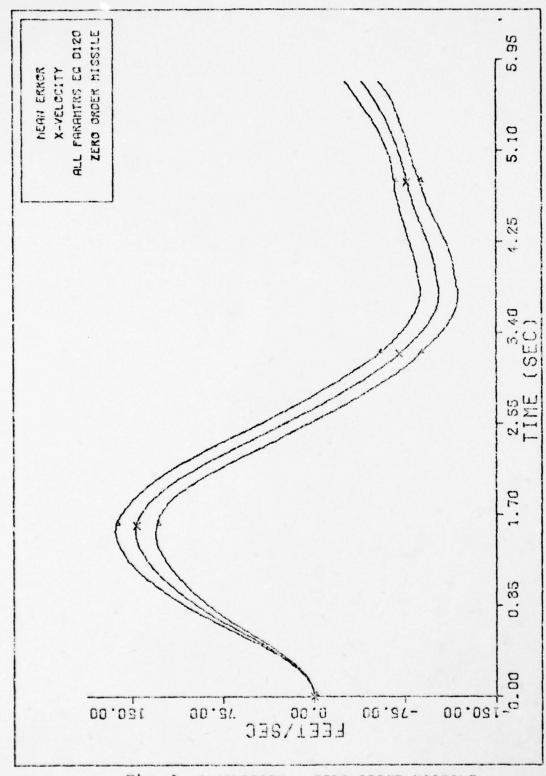


Fig. 1. X-VELOCITY ZERO ORDER MISSILE

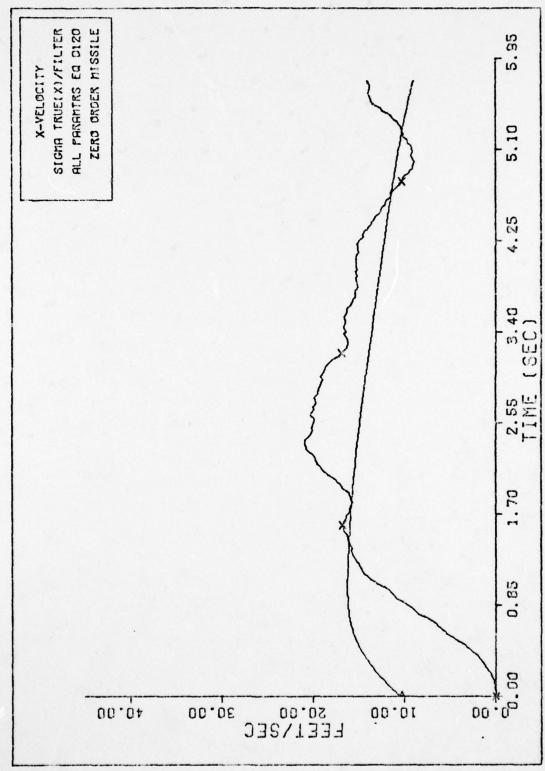


Fig. 2. X-VELOCITY SIGMAS ZERO ORDER

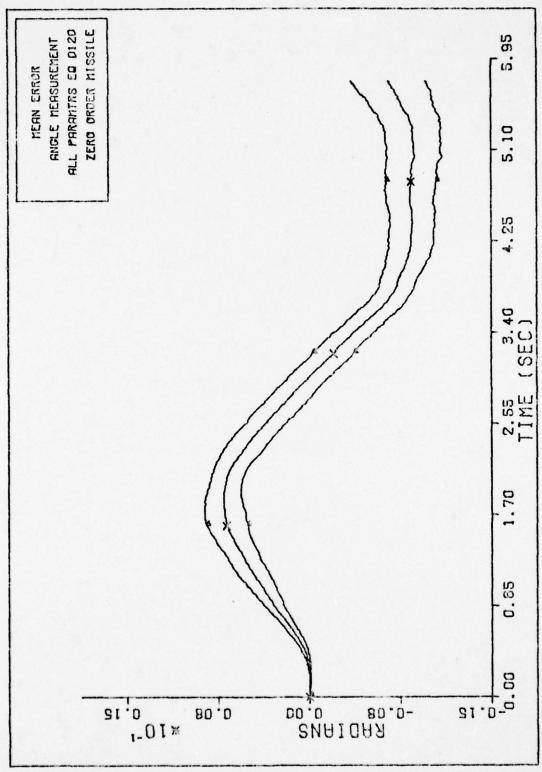


Fig. 3. ANGLE MEASUREMENT ZERO DRDER MISSILE

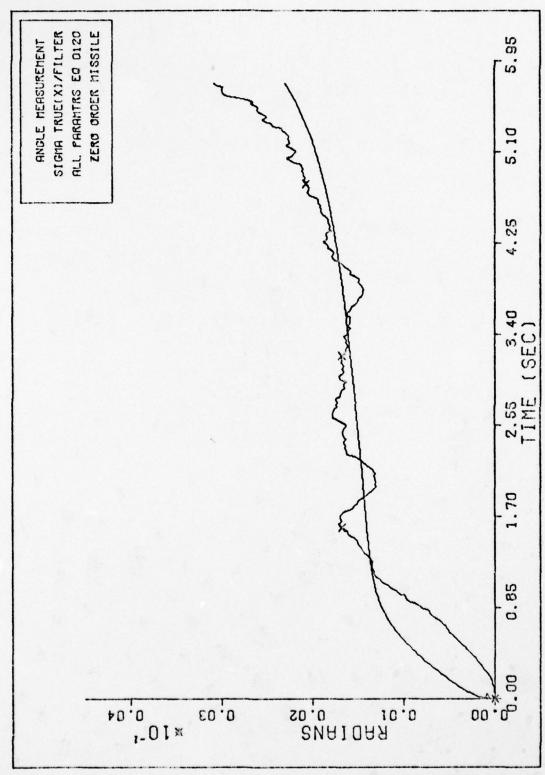


Fig. 4. ANGLE MERSUREMENT SIGMAS ZERO ORDER

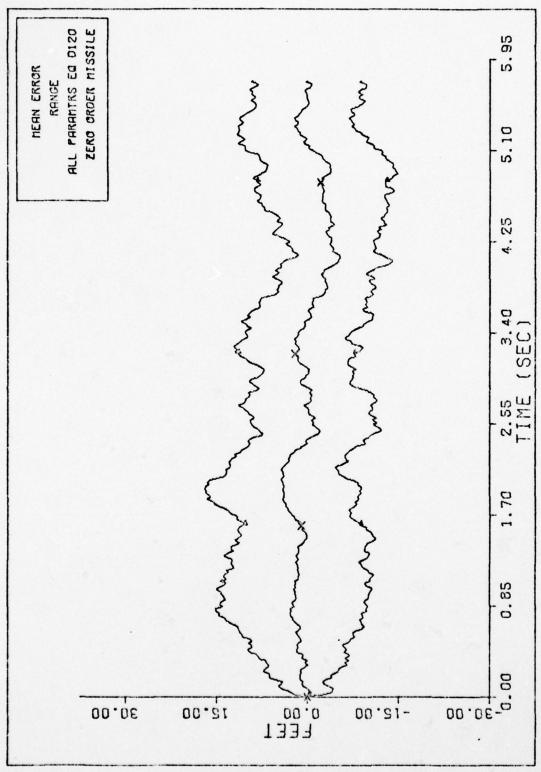


Fig. 5. RANGE ZERO DRDER MISSILE

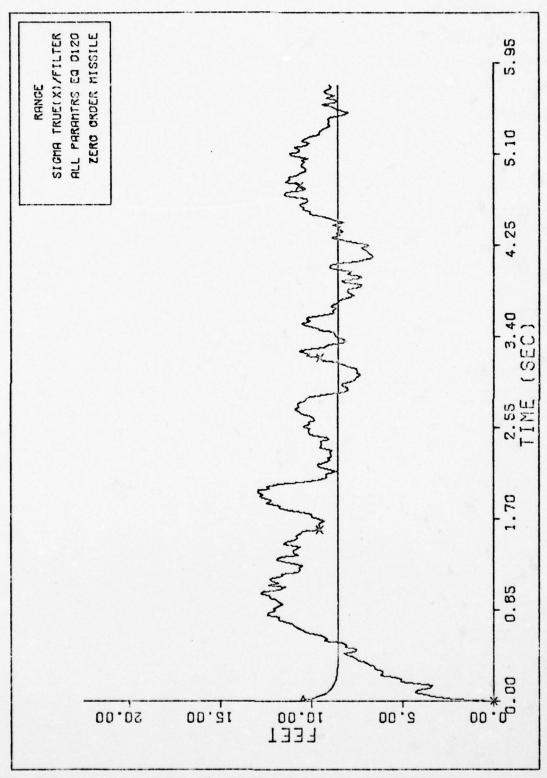


Fig. 6. RANGE SIGMAS ZERG ORDER

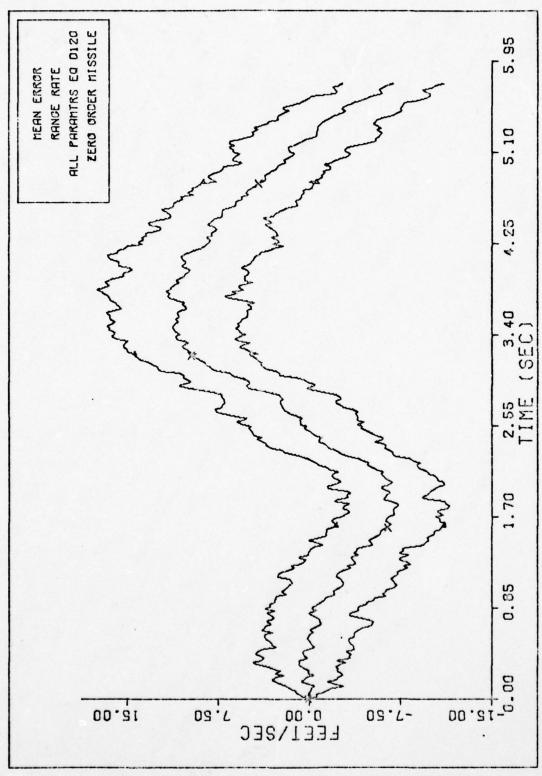


Fig. 7. RANGE RATE ZERO ORDER MISSILE

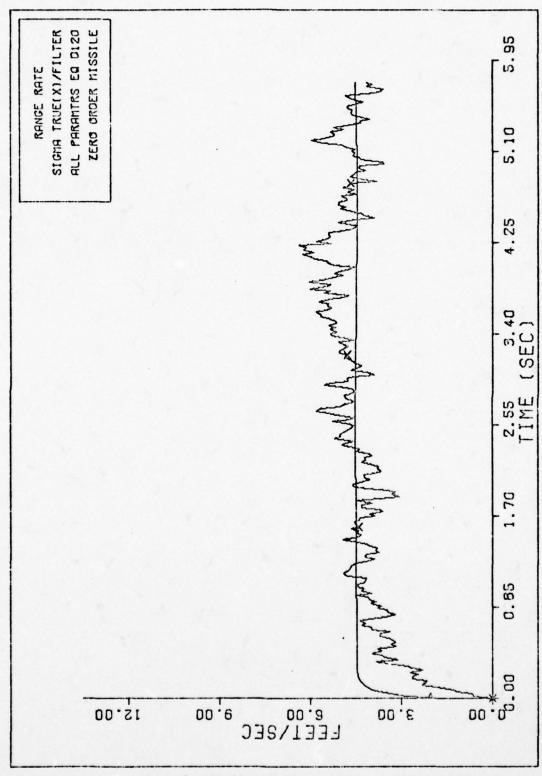


Fig. 8. RANGE RATE SIGMAS ZERO ORDER

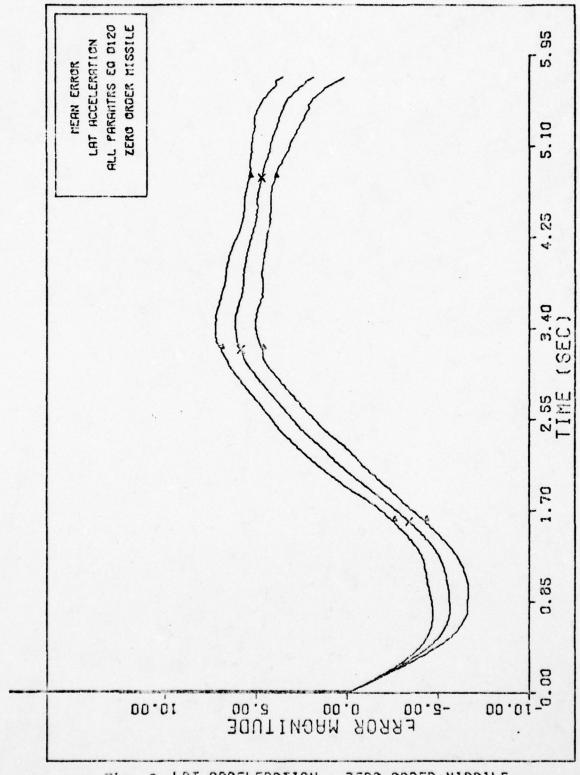


Fig. 9. LAT ACCELERATION ZERO ORDER MISSILE

### Third Order Missile Filter

The initial state estimates and the tuning parameters for this case are

$$v_{mx}^{I}(0) = 1225.7 \text{ fps}$$
 $\theta(0) = 4.363345 \text{ radians}$ 
 $R(0) = 10000. \text{ feet}$ 
 $\dot{R}(0) = -2122. \text{ fps}$ 
 $a_{L}(0) = 0.$ 
 $n(0) = 4.5$ 
 $\tau_{f}(0) = N/A$ 
 $M/S(0) = 29.197 \text{ slugs/ft}^{2}$ 

$$\underline{\mathbf{R}} = \begin{bmatrix} \overline{\mathbf{4}} \cdot \mathbf{E} - 5 & 0 & 0 & 0 \\ 0 & 500 & 0 & 0 \\ 0 & 0 & 100 & 0 \end{bmatrix}$$

This design was used as an attempt in establishing a benchmark. The filter model included the exact structure of the autopilot (found by assuming constant coefficients as described in Chapter III). Chapter III lists six sets of coefficients found at various times over the high-g scenario. The plots that follow were generated using the coefficients for t=0. This set of coefficients reduced the initial transients in the filter. This can be seen by comparing the plots of the fourth order missile filter which use three sets of autopilot transfer function coefficients to demonstrate this point.

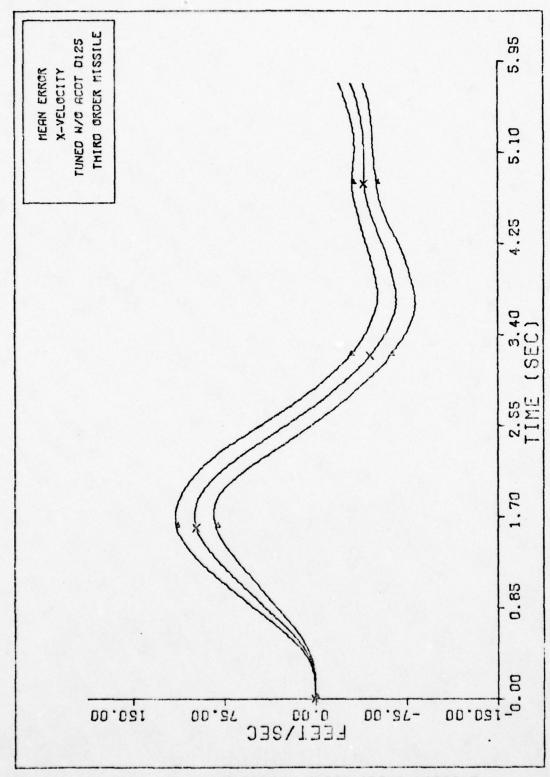


Fig. 10. X-VELOCITY THIRD ORDER MISSILE

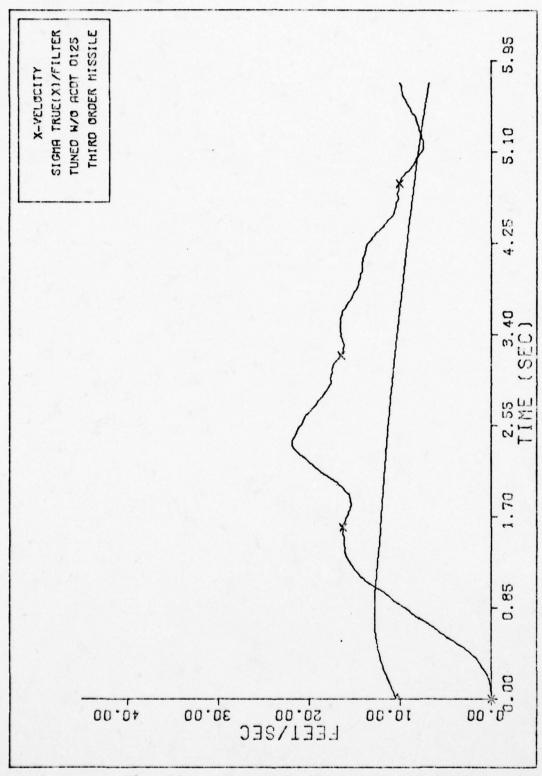


Fig. 11. X-VELOCITY SIGMAS THIRD ORDER

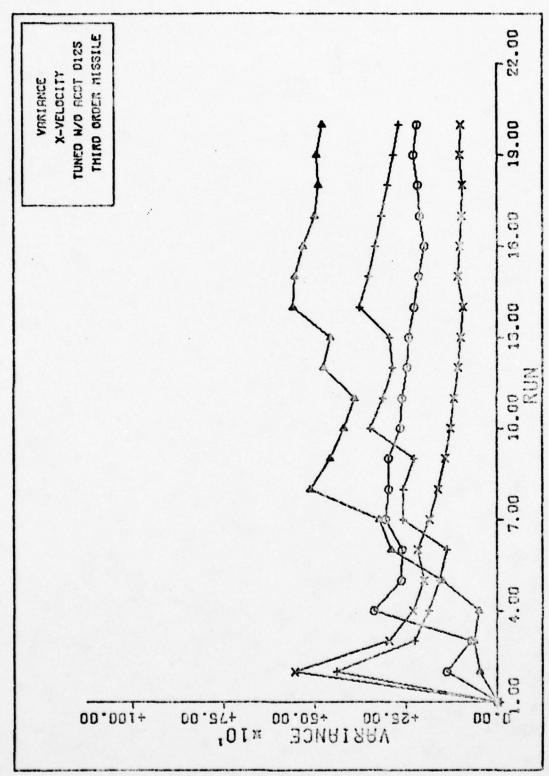


Fig. 12. VARIANCE CONVERGENCE

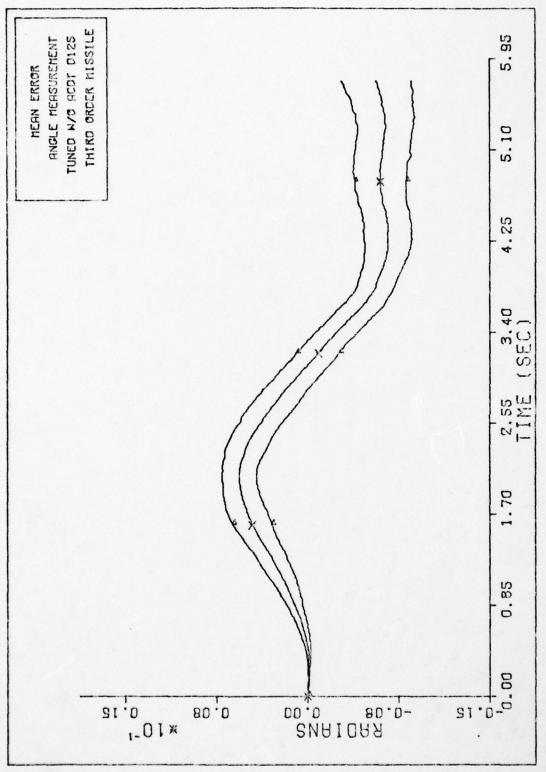


Fig. 13. ANGLE MEASUREMENT THIRD ORDER MISSILE

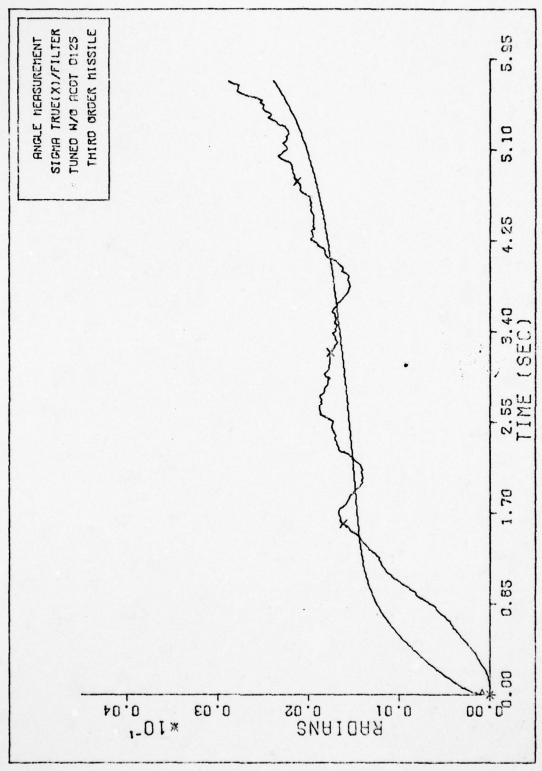


Fig. 14. ANOLE MEASUREMENT SIGMAS THIRD ORDER

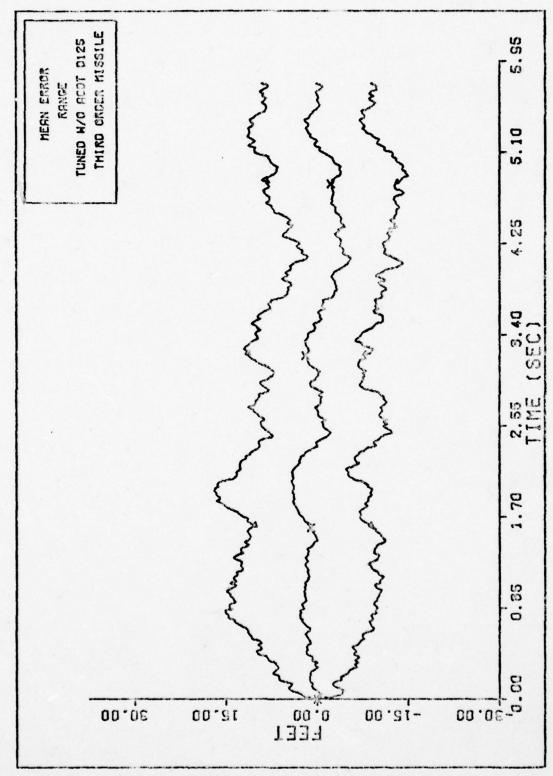


Fig. 15. RANGE THIRD ORDER HISSILE

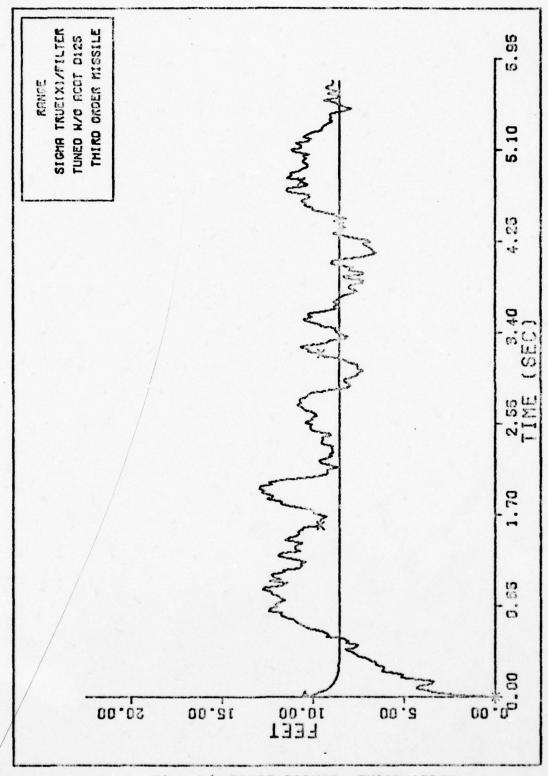


Fig. 16. RANGE SIGMAS THIRD GROER

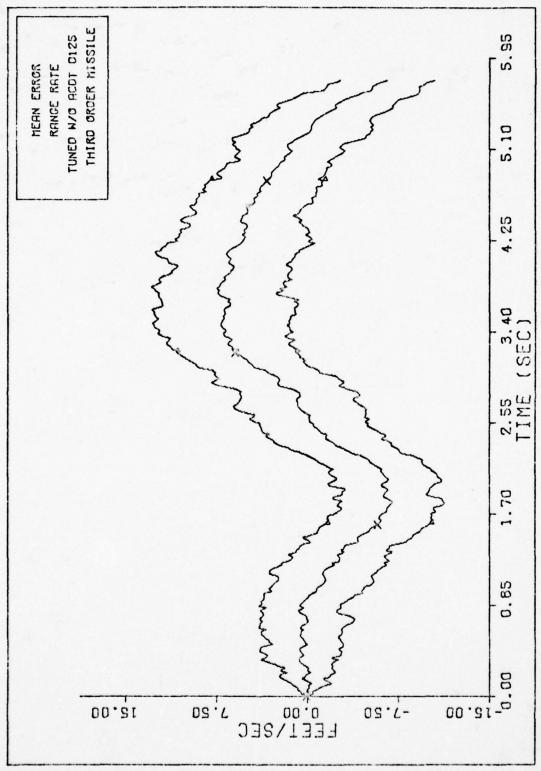


Fig. 17. RANGE RATE THIRD ORDER MISSILE

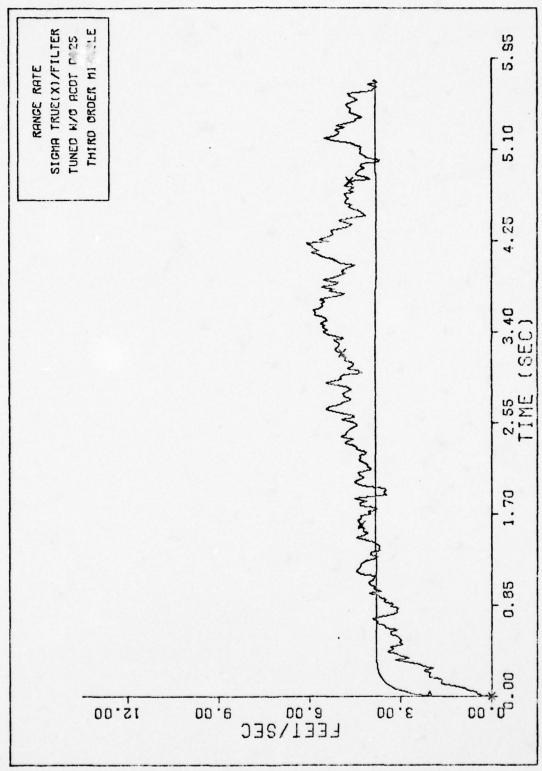


Fig. 18. RANGE RATE SIGMAS THIRD ORDER

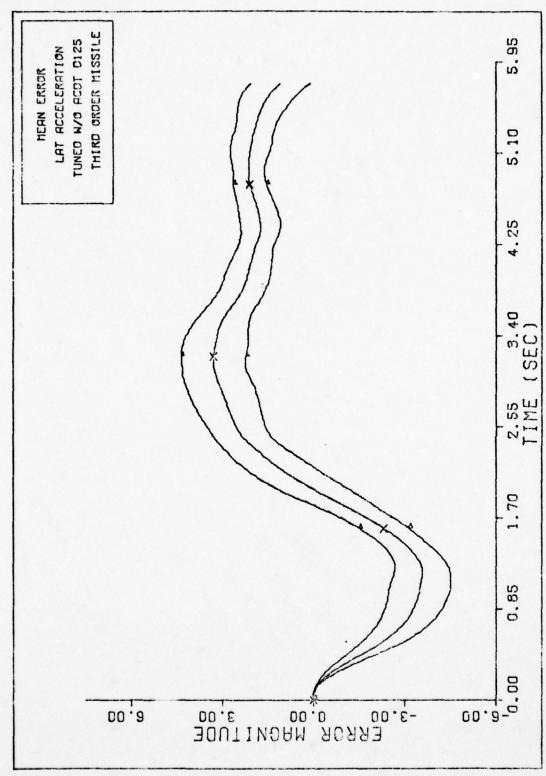


Fig. 19. LAT ACCELERATION THIRD ORDER MISSILE

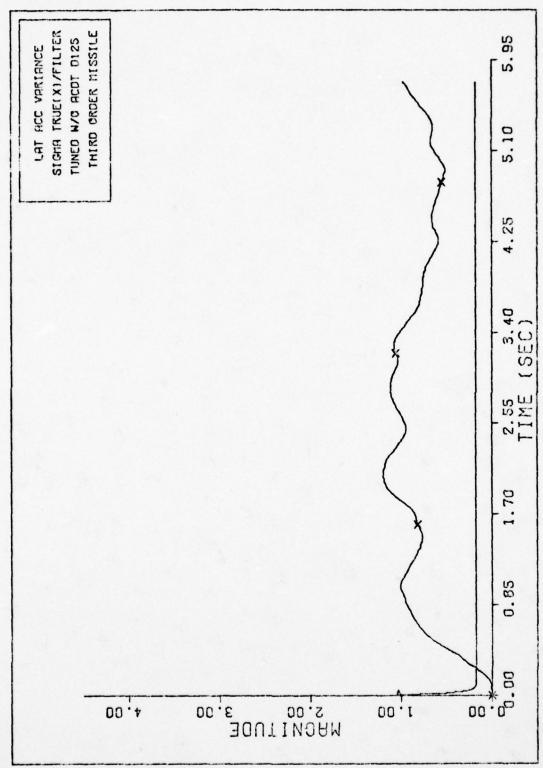


Fig. 20. LAT ACCELERATION SIGMAS THIRD ORDER

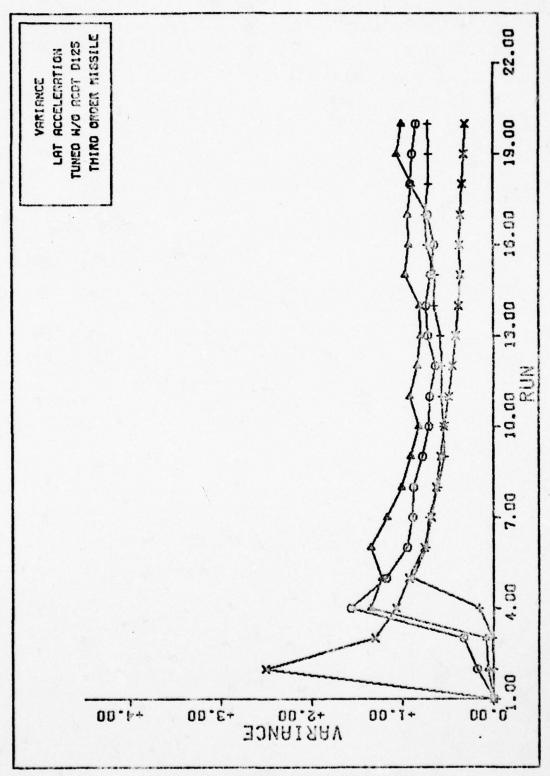


Fig. 21. VARIANCE CONVERGENCE

#### Benchmark

The initial state estimates and the tuning parameters for this case are

$$V_{mx}^{I}(0) = 1225.7 \text{ fps}$$
 $\dot{\theta}(0) = 4.363345 \text{ radians}$ 
 $R(0) = 10000. \text{ feet}$ 
 $\dot{R}(0) = -2122. \text{ fps}$ 
 $a_{L}(0) = 0.$ 
 $n(0) = 4.5$ 
 $\tau_{f}(0) = N/A$ 
 $M/S(0) = 29.197 \text{ slugs/ft}^{2}$ 

$$\underline{\mathbf{R}} = \begin{bmatrix} 4.E-5 & 0. & 0. \\ 0. & 500. & 0. \\ 0. & 0. & 100. \end{bmatrix}$$

A complete description of the ad hoc design, used in generating the following plots, can be found in Chapter III. Basically, the true commanded acceleration of the missile was assumed known perfectly and passed directly to the autopilot in the filter model. The autopilot transfer function used the coefficients for t=0 (refer to Chapter III). These plots were considered as benchmarks for the five dynamic states of the fundamental filter design.

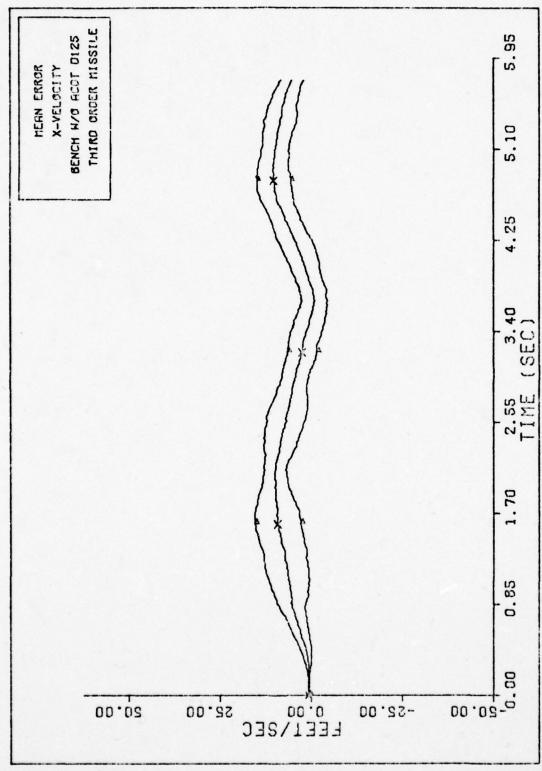


Fig. 22. X-VELOCITY THIRD ORDER MISSILE

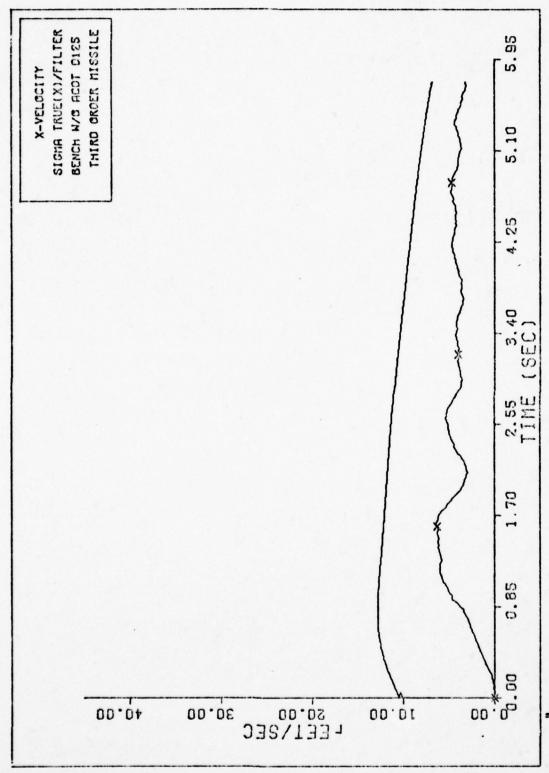


Fig. 23. Y-VELOCITY SIGMAS THIRD CREER

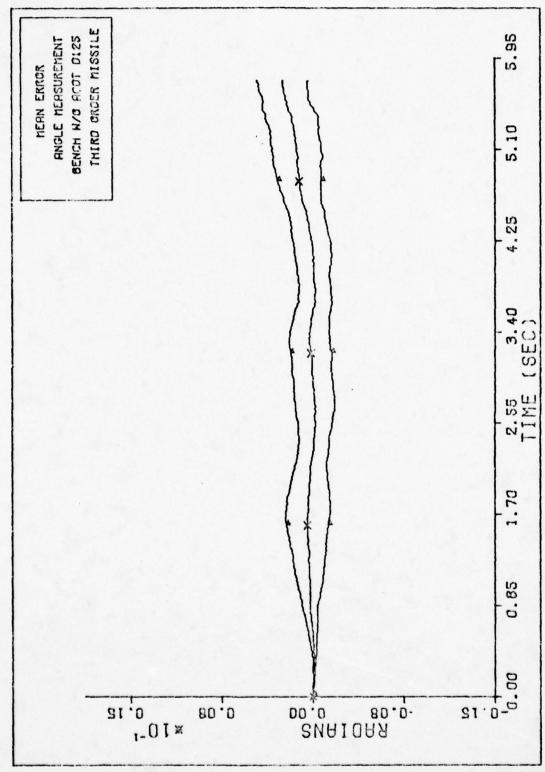


Fig. 24. ANGLE MEASUREMENT THIRD DRDER MISSILE

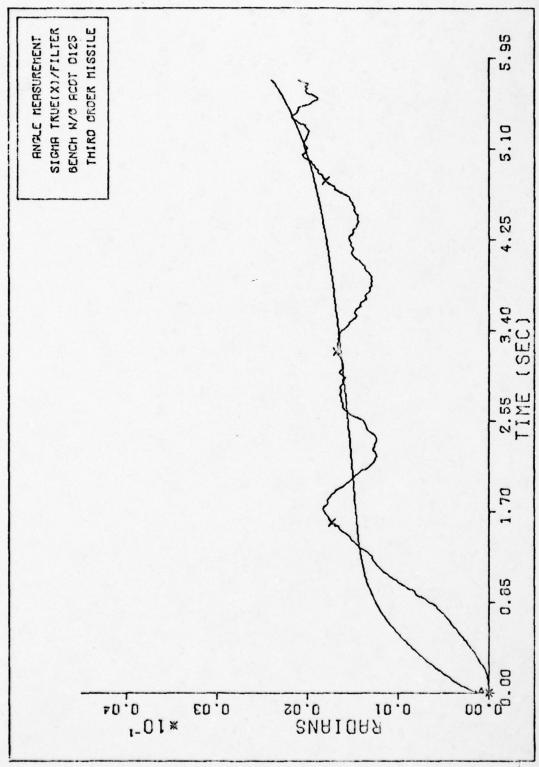


Fig. 25. ANG! E MEASUREMENT SIGMAS THIRD ORDER

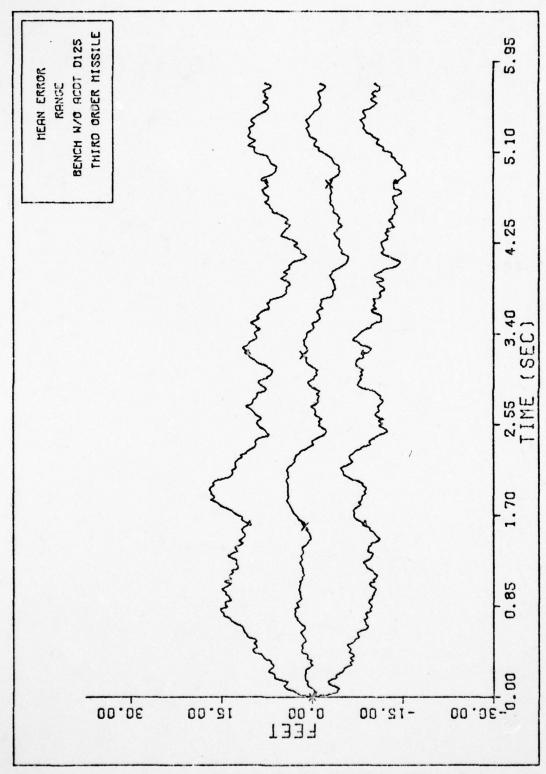


Fig. 26. RANGE THIRD ORDER MISSILE

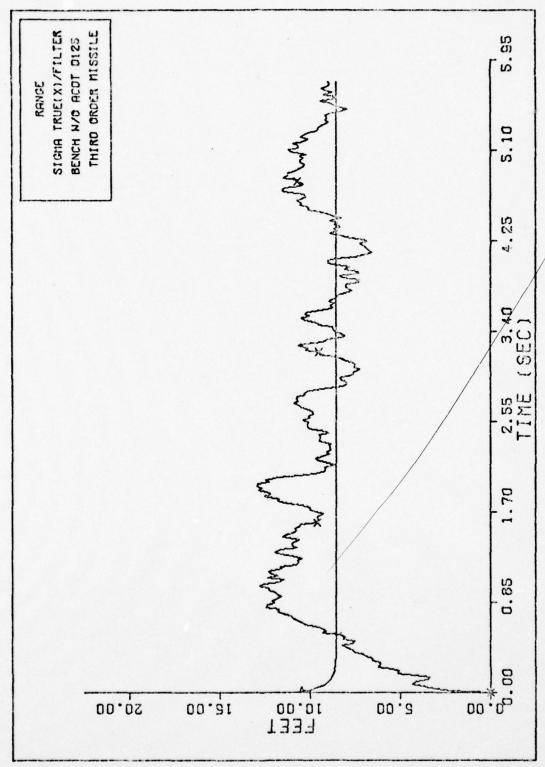


Fig. 27. RANGE SIGMAS THIRD ORDER

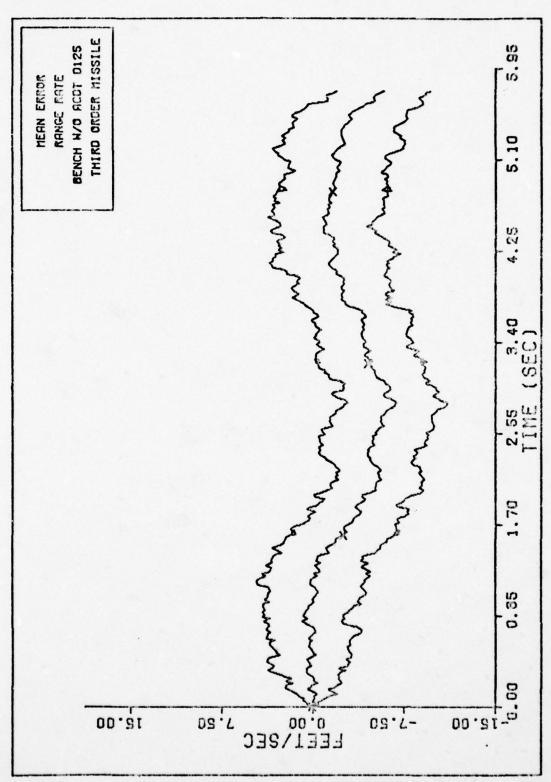


Fig. 28. RANGE RATE THIRD ORDER MISSILE

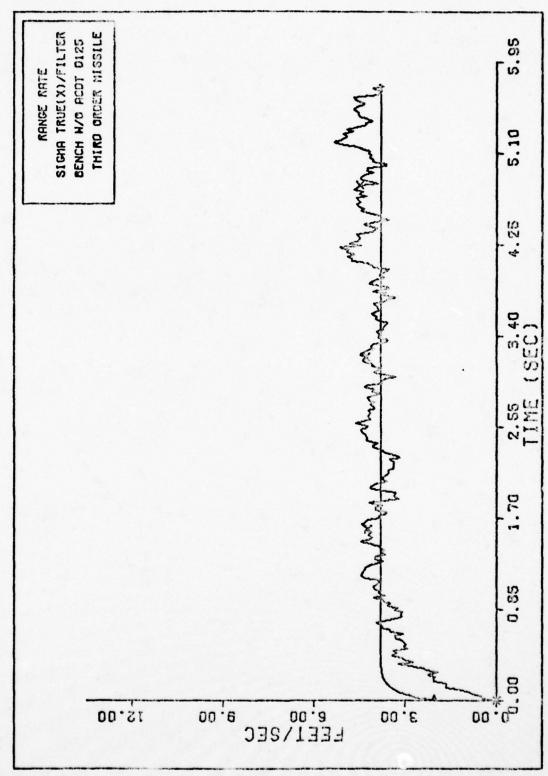


Fig. 29. RANGE RATE SIGMAS THIRD ORDER

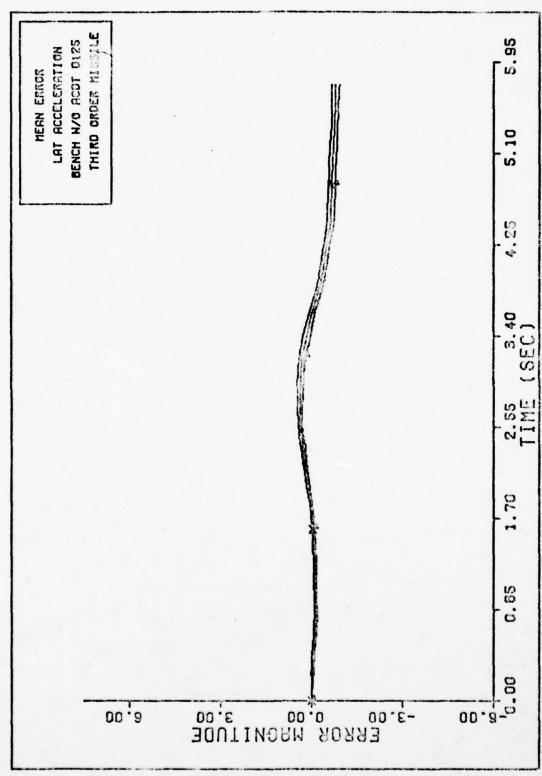


Fig. 30. LAT ACCELERATION THIRD ORDER MISSILE

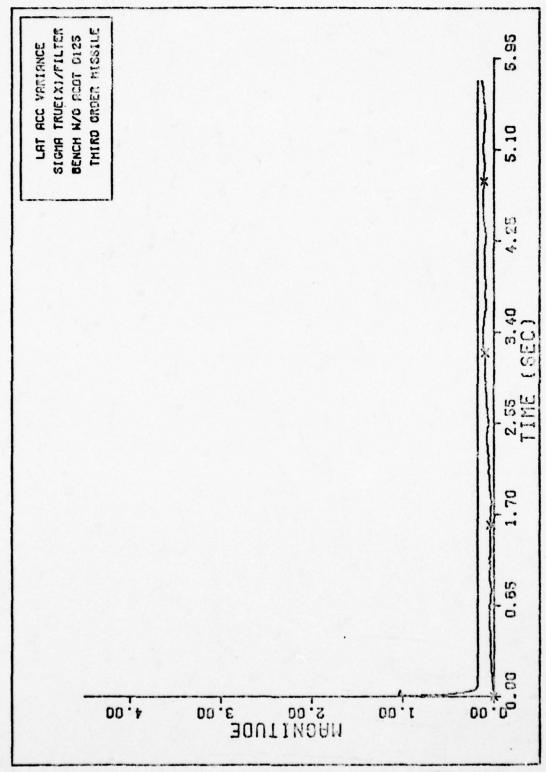


Fig. 31. LAT ACCELERATION SIGMAS THIRD ORDER

## Fourth Order Missile Filter (A/P at 0 sec)

The initial state estimates and the tuning parameters for this case are

$$v_{mx}^{I}(0) = 1225.7 \text{ fps}$$
 $\dot{\theta}(0) = 4.363345 \text{ radians}$ 
 $R(0) = 10000. \text{ feet}$ 
 $\dot{R}(0) = -2122. \text{ fps}$ 
 $a_{L}(0) = 0.$ 
 $n(0) = 4.5$ 
 $\tau_{f}(0) = N/A$ 
 $M/S(0) = 29.197 \text{ slugs/ft}^{2}$ 

$$\underline{R} = \begin{bmatrix} \overline{4.E-5} & 0. & 0. \\ 0. & 500. & 0. \\ 0. & 0. & 100. \end{bmatrix}$$

These plots were generated by the fourth order filter with the coefficients of the autopilot transfer function determined at t=0. These plots were compared to those results of the fourth order filter using autopilot coefficients for t=3 and t=5. The intent was to determine if there was any distinguishable difference in filter performance for the various sets of autopilot coefficients. A complete description of the filter autopilot and its coefficients can be found in Chapter III.

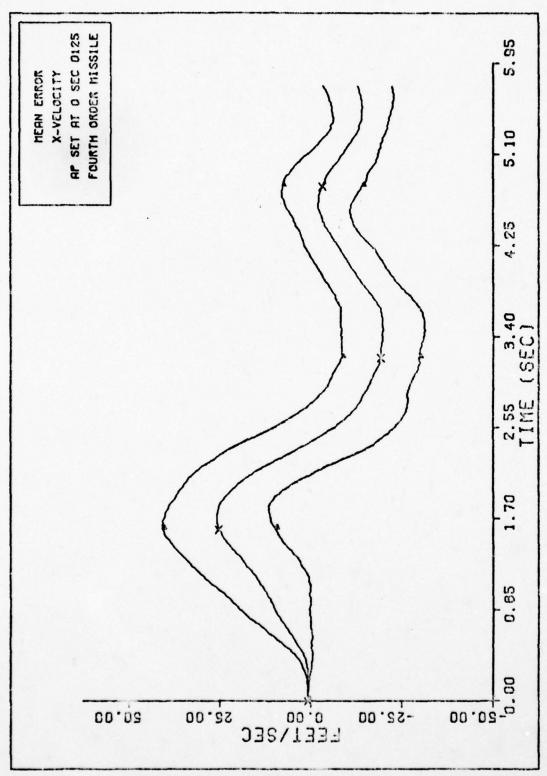


Fig. 32. X-VELOCITY FOURTH ORDER MISSILE

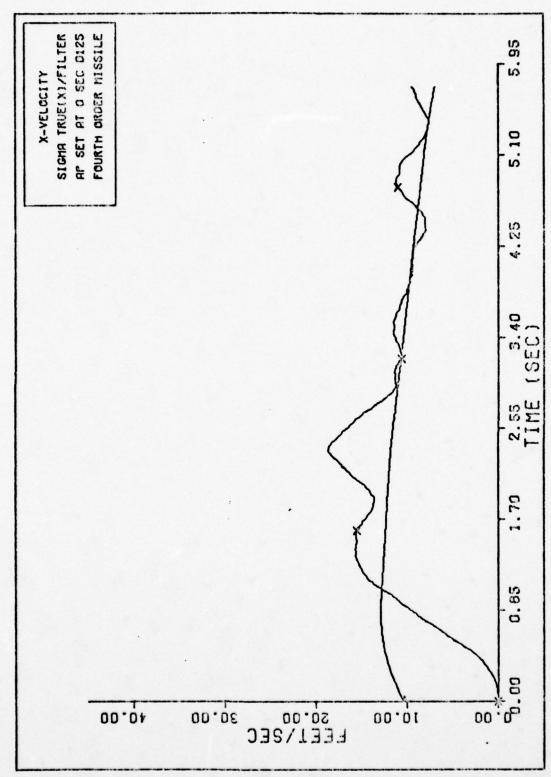


Fig. 33. X-VELOCITY SIGMAS FOURTH GROER

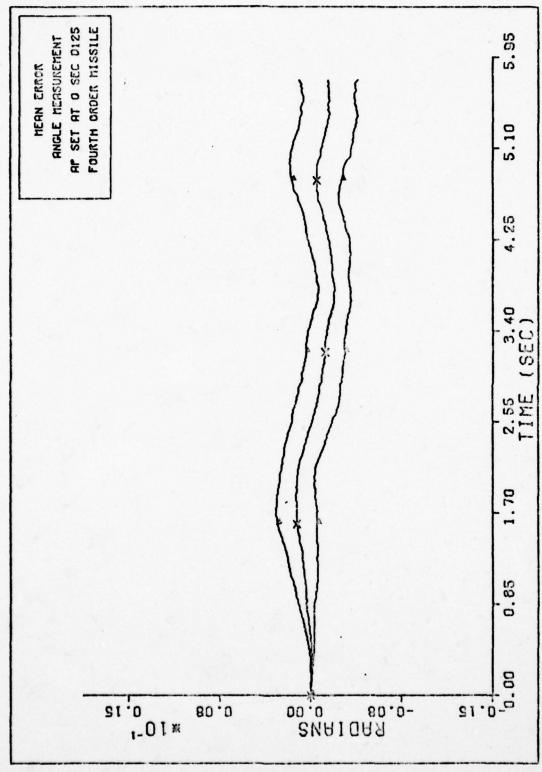


Fig. 34. ANGLE MEASUREMENT FOURTH ORDER MISSILE

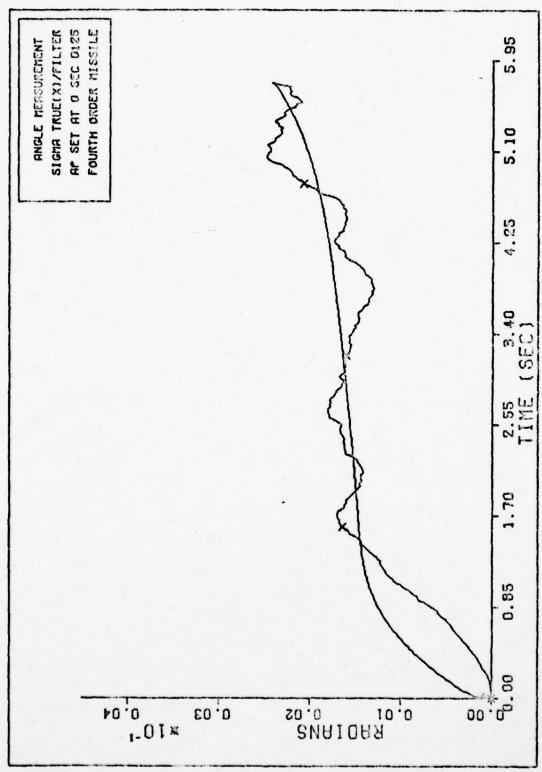


Fig. 35. ANGLE MEASUREMENT SIGNAS FOURTH ORDER

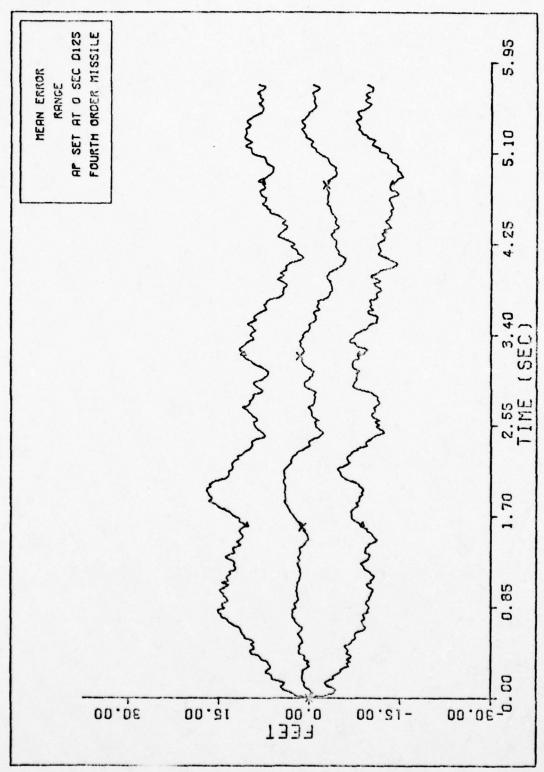


Fig. 36. RANGE FOURTH ORDER MISSILE

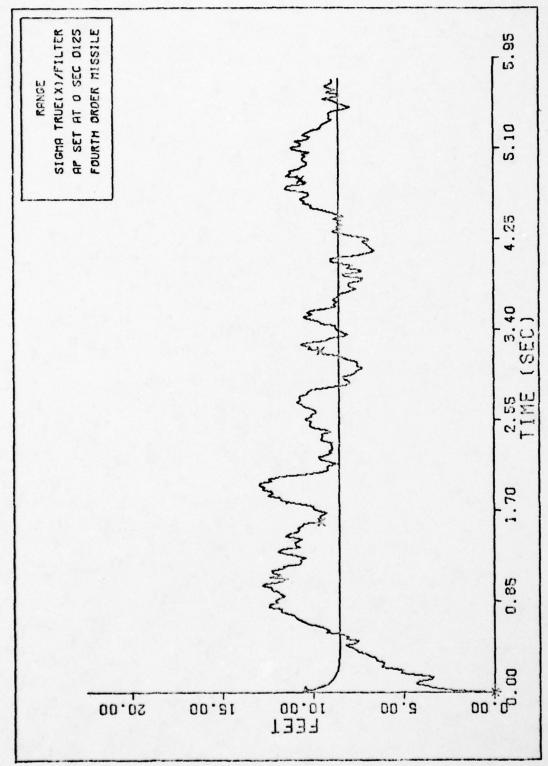


Fig. 37. RANGE SIGMAS FOURTH ORDER

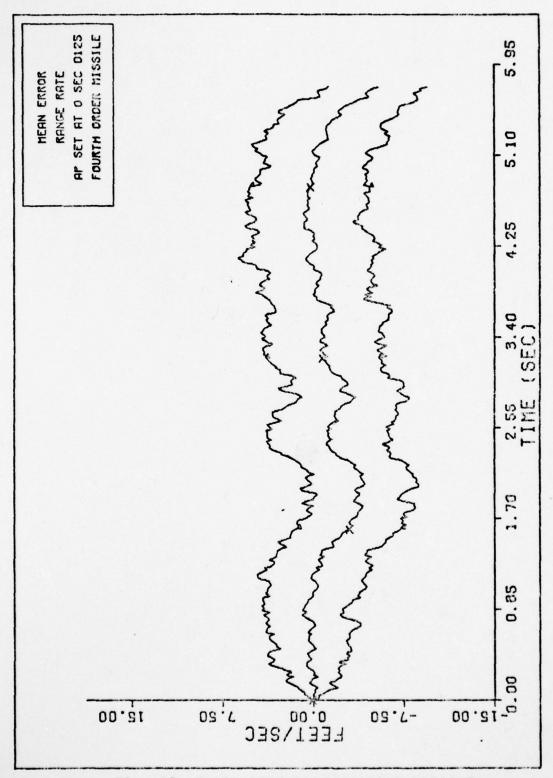


Fig. 38. RANGE RATE FOURTH ORDER MISSILE

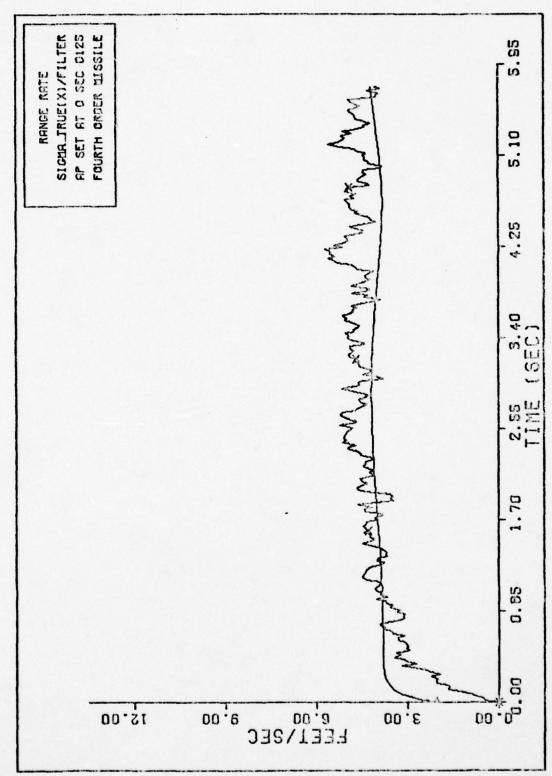


Fig. 39. RANGE RATE SIGMAS FOURTH ORDER

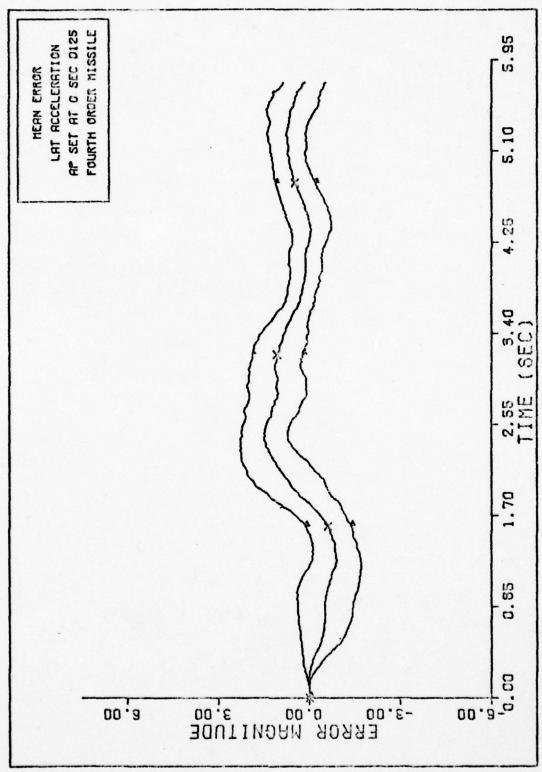


Fig. 40. LRT RCCELERATION FOURTH ORDER MISSILE

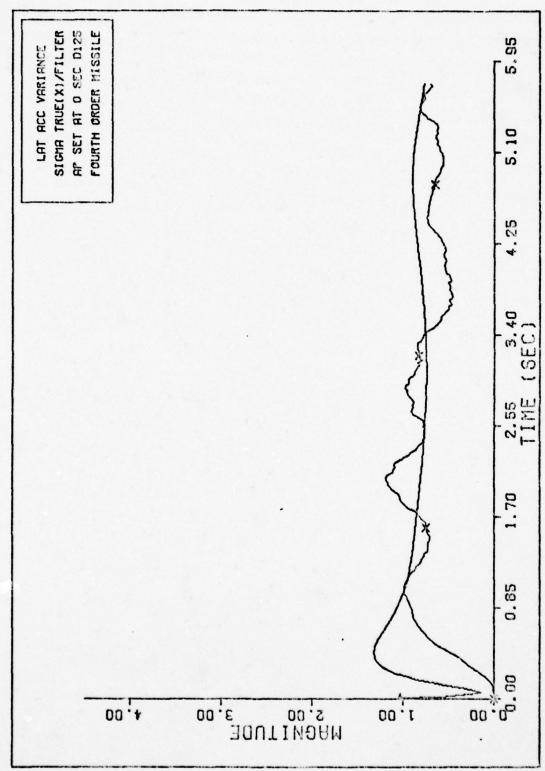


Fig. 41. LAT ACCELERATION SIGMAS FOURTH ORDER

## Fourth Order Missile Filter (A/P at 3 sec)

The initial state estimates and the tuning parameters for this case are

$$V_{mx}^{I}(0) = 1225.7 \text{ fps}$$
 $\theta_{I}(0) = 4.363345 \text{ radians}$ 
 $R(0) = 10000. \text{ feet}$ 
 $\dot{R}(0) = -2122. \text{ fps}$ 
 $a_{I,I}(0) = 0.$ 
 $n(0) = 4.5$ 
 $\tau_{f}(0) = N/A$ 
 $M/S(0) = 29.197 \text{ slugs/ft}^{2}$ 

$$\underline{\mathbf{R}} = \begin{bmatrix} \overline{\mathbf{4}} \cdot \mathbf{E} - 5 & 0 & 0 & 0 \\ 0 & 500 & 0 & 0 \\ 0 & 0 & 100 & 0 \end{bmatrix}$$

These plots were generated by the fourth order filter with the coefficients of the autopilot transfer function determined at t=3. These plots were compared to those results of the fourth order filter using autopilot coefficients for t=0 and t=5. The intent was to determine if there was any distinguishable difference in filter performance for the various sets of autopilot coefficients. A complete description of the filter autopilot and its coefficients can be found in Chapter III.

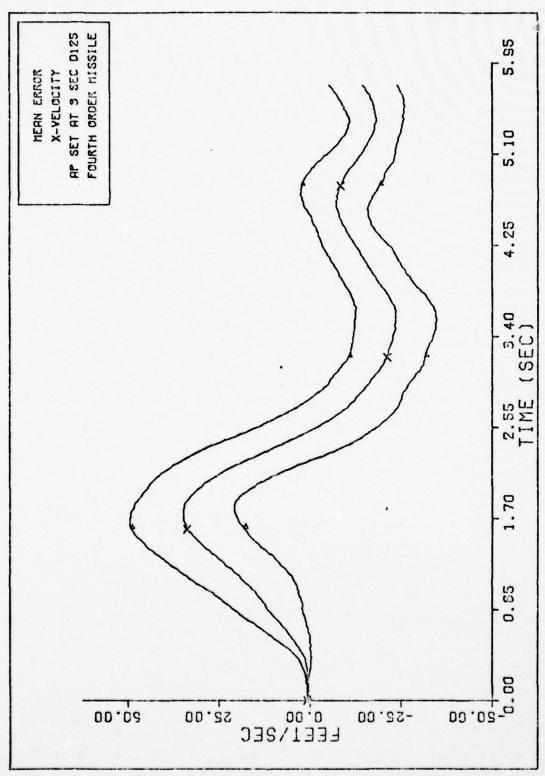


Fig. 42. X-VELOCITY FOURTH ORDER MISSILE

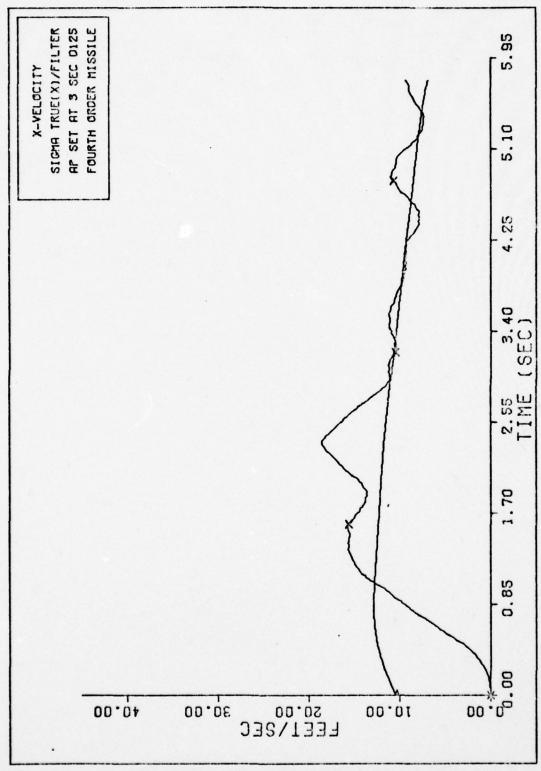


Fig. 43. X-VELOCITY SIGMAS FOURTH ORDER

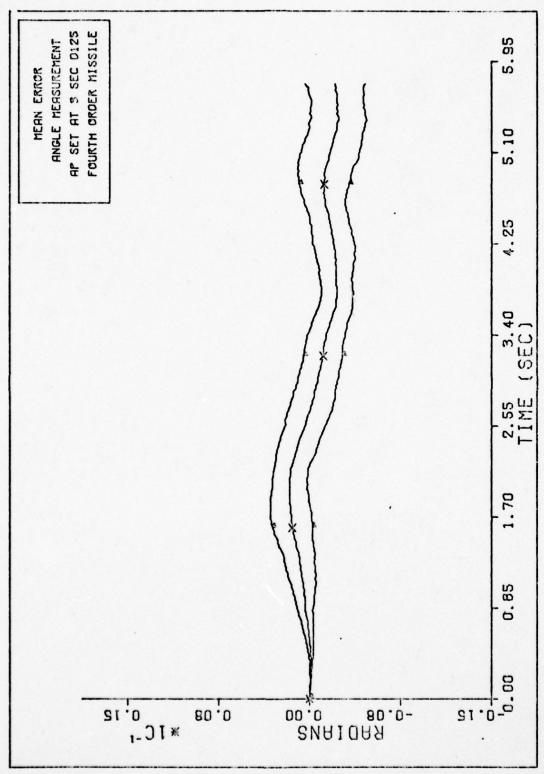


Fig. 44. ANGLE MEASUREMENT FOURTH ORDER MISSILE

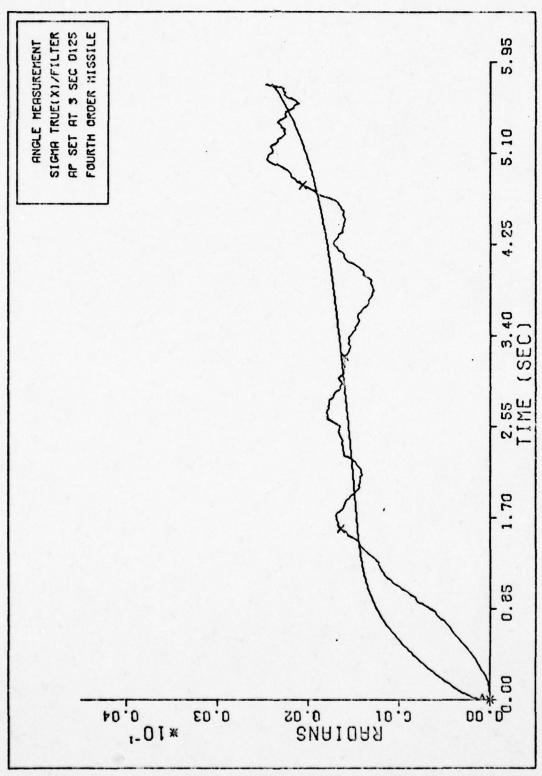


Fig. 45. ANGLE MEASUREMENT SIGMAS FOURTH ORDER

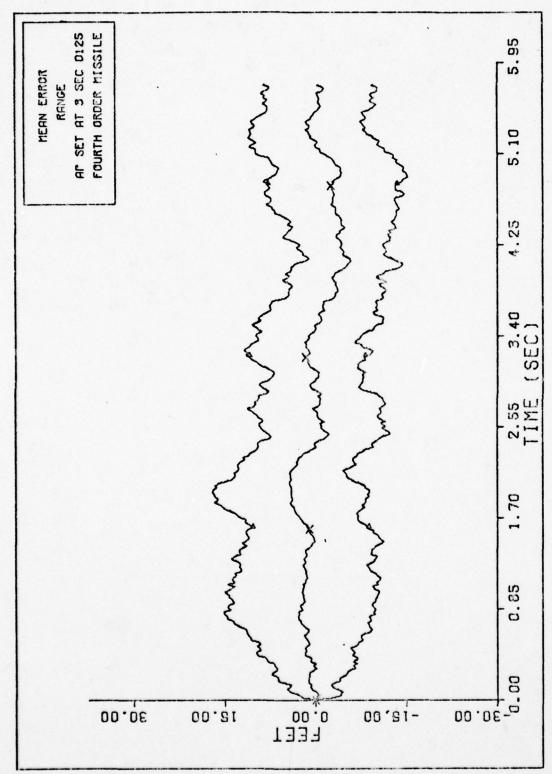


Fig. 46. RANGE FOURTH ORDER MISSILE

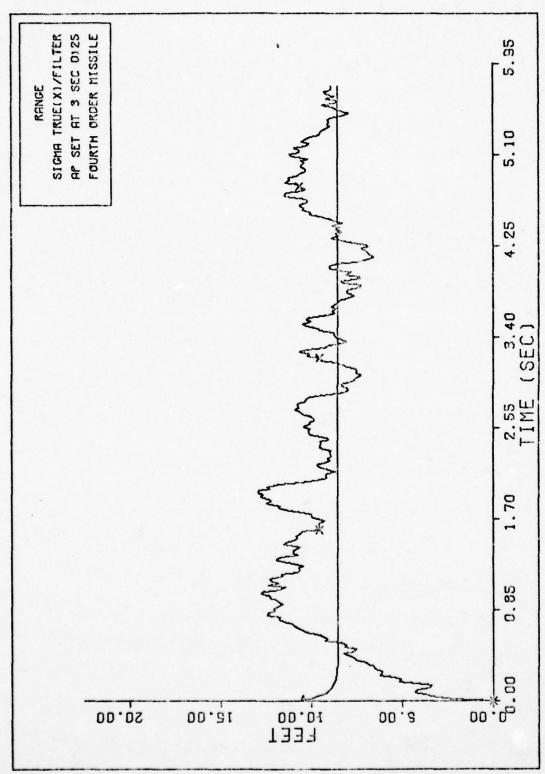


Fig. 47. RANGE SIGMAS FOURTH ORDER

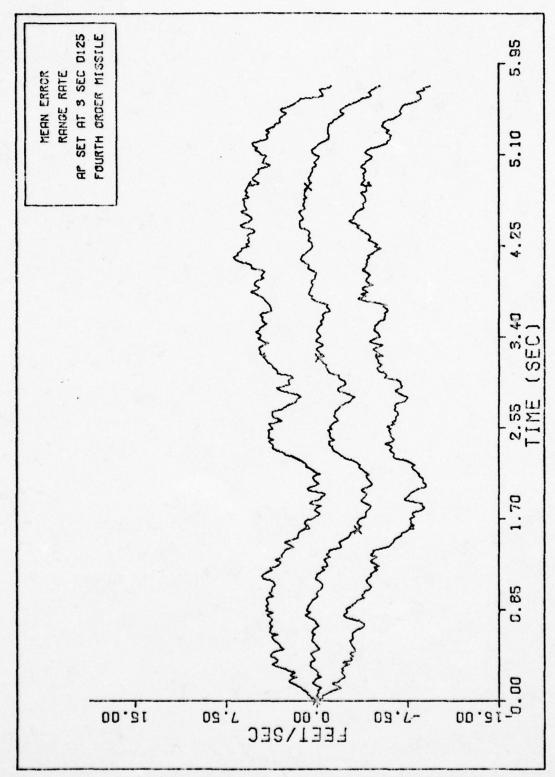


Fig. 48. RANGE RATE FOURTH DRDER MISSILE

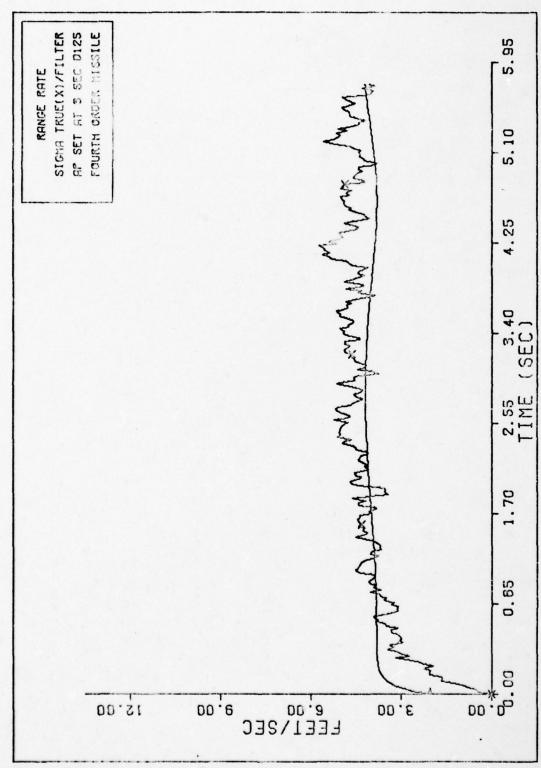


Fig. 49. RANGE RATE SIGMAS FOURTH ORDER

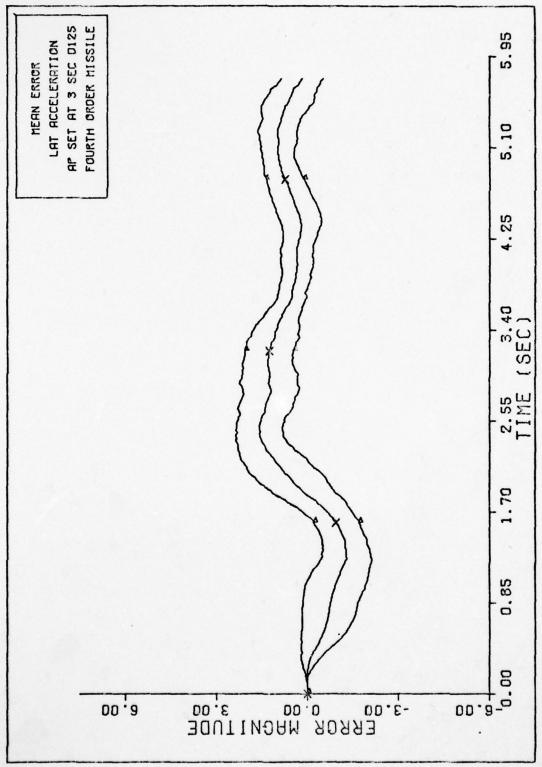


Fig. 50. LAT ACCELERATION FOURTH ORDER MISSILE

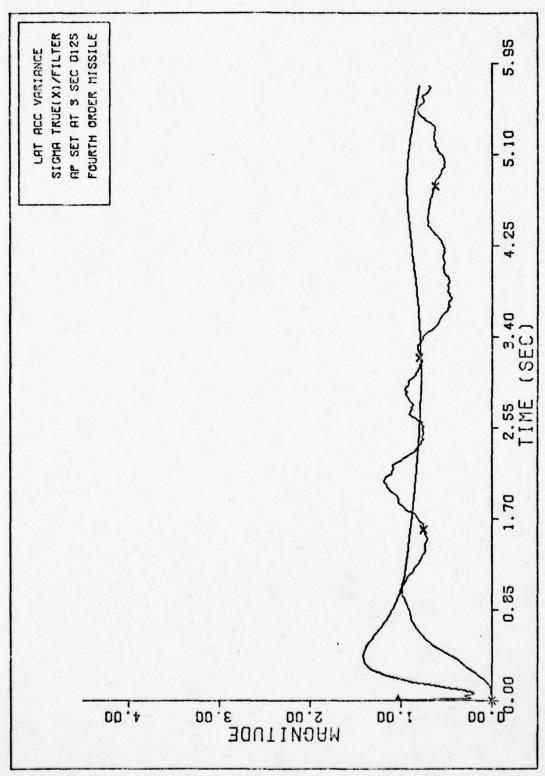


Fig. 51. LAT ACCELERATION SIGMAS FOURTH ORDER

## Fourth Order Missile Filter (A/P at 5 sec)

The initial state estimates and the tuning parameters for this case are

$$V_{mx}^{I}(0) = 1225.7 \text{ fps}$$
 $\dot{\theta}_{I}(0) = 4.363345 \text{ radians}$ 
 $R(0) = 10000. \text{ feet}$ 
 $\dot{R}(0) = -2122. \text{ fps}$ 
 $a_{L}(0) = 0.$ 
 $n(0) = 4.5$ 
 $T_{f}(0) = N/A$ 
 $M/S(0) = 29.197 \text{ slugs/ft}^{2}$ 

$$\underline{\mathbf{R}} = \begin{bmatrix} \overline{4} \cdot \mathbf{E} - 5 & 0 & 0 & 0 \\ 0 & 500 & 0 & 0 \\ 0 & 0 & 100 & 0 \end{bmatrix}$$

These plots were generated by the fourth order filter with the coefficients of the autopilot transfer function determined at t=5. These plots were compared to those results of the fourth order filter using autopilot coefficients for t=0 and t=3. The intent was to determine if there was any distinguishable difference in filter performance for the various sets of autopilot coefficients. A complete description of the filter autopilot and its coefficients can be found in Chapter III.

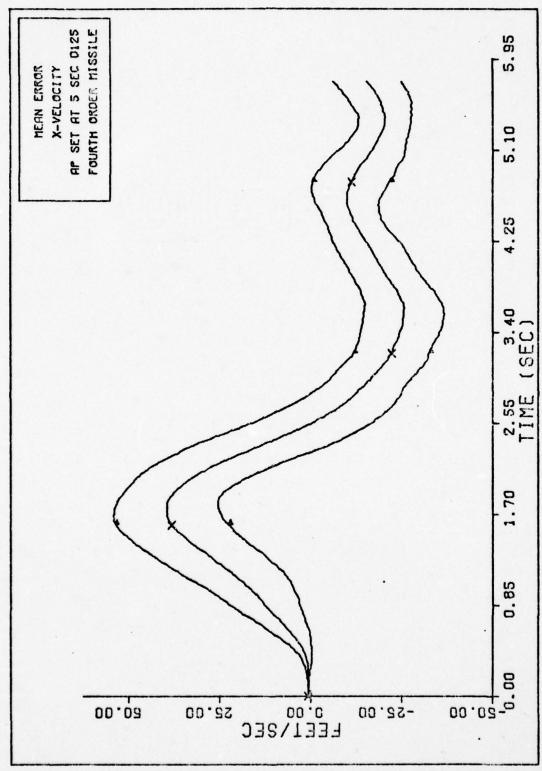


Fig. 52. X-VELOCITY FOURTH ORDER MISSILE

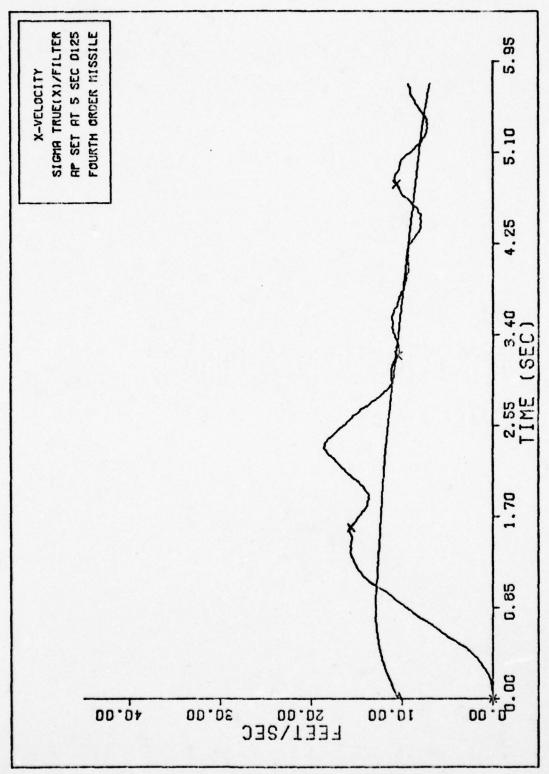


Fig. 53. X-VELOCITY SIGMAS FOURTH ORDER

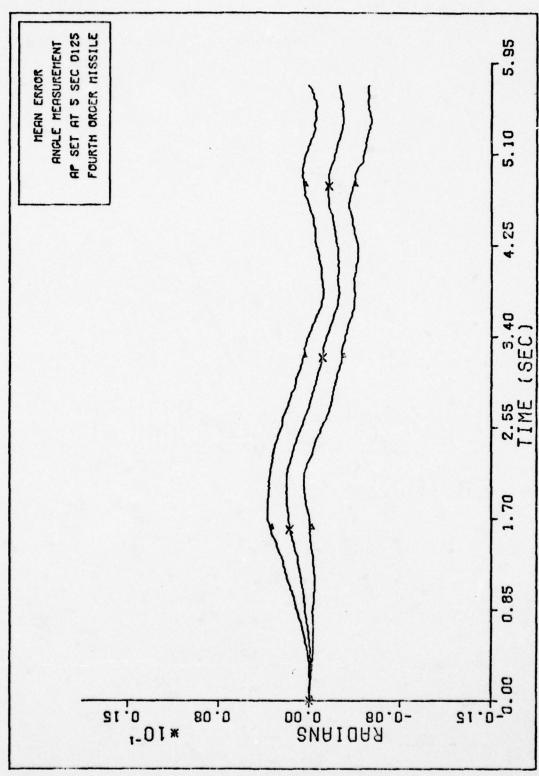


Fig. 54. ANGLE MEASUREMENT FOURTH DRDER MISSILE

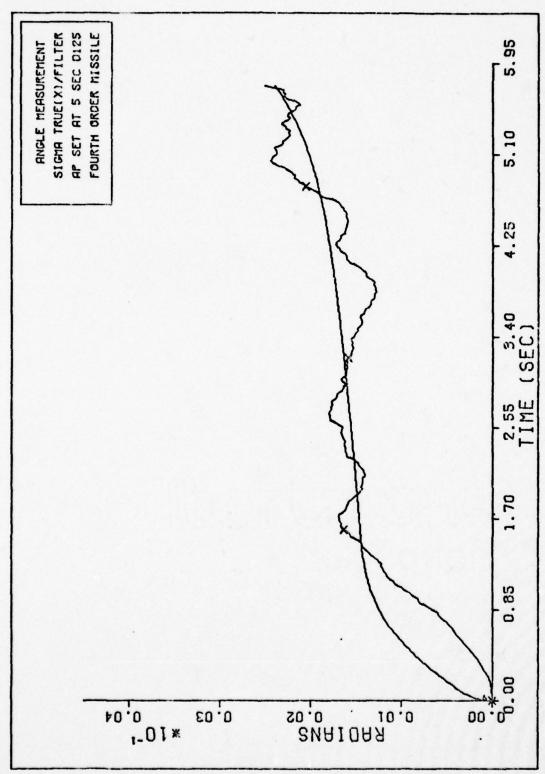


Fig. 55. ANGLE MERSUREMENT SIGMAS FOURTH ORDER

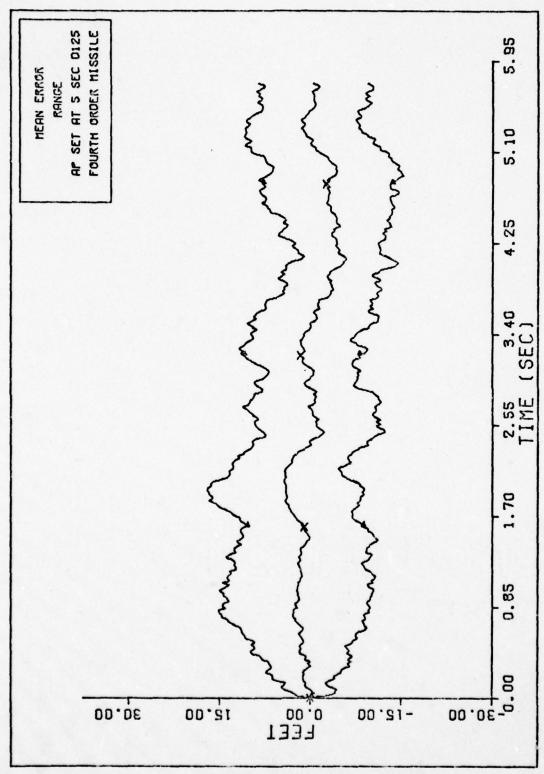


Fig. 56. RANGE FOURTH ORDER MISSILE

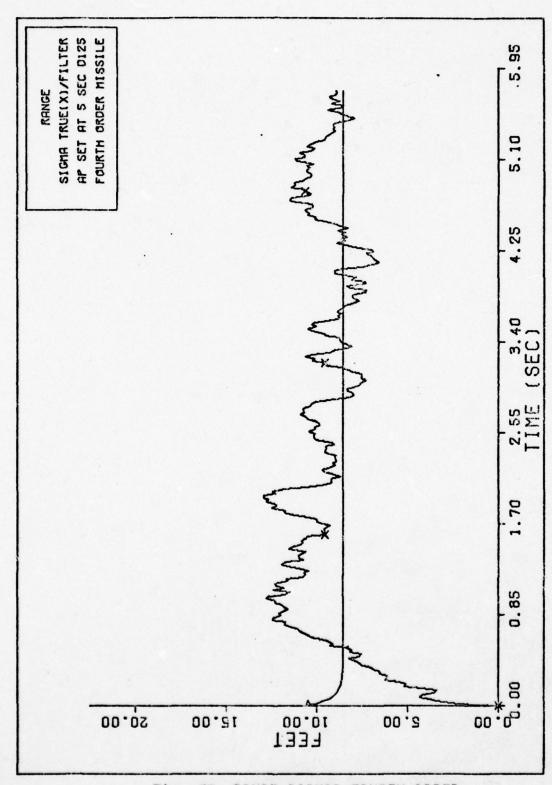


Fig. 57. RANGE SIGMAS FOURTH ORDER

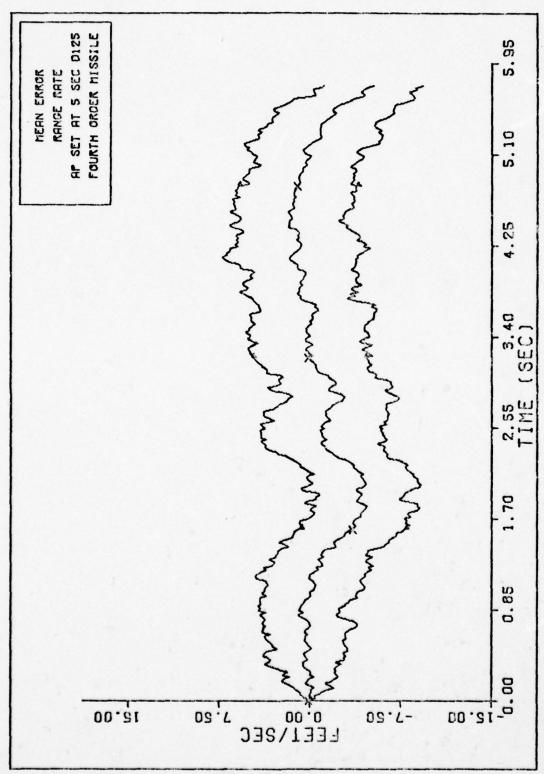


Fig. 58. RANGE RATE FOURTH ORDER MISSILE

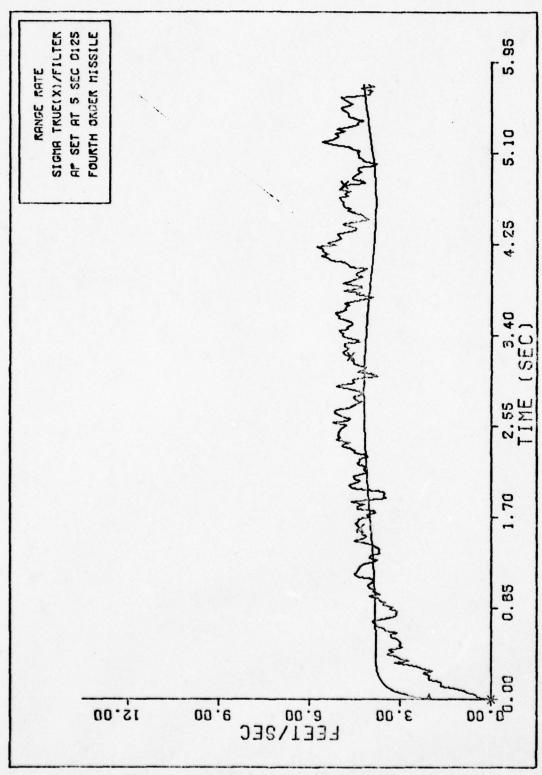


Fig. 59. RANGE RATE SIGMAS FOURTH ORDER

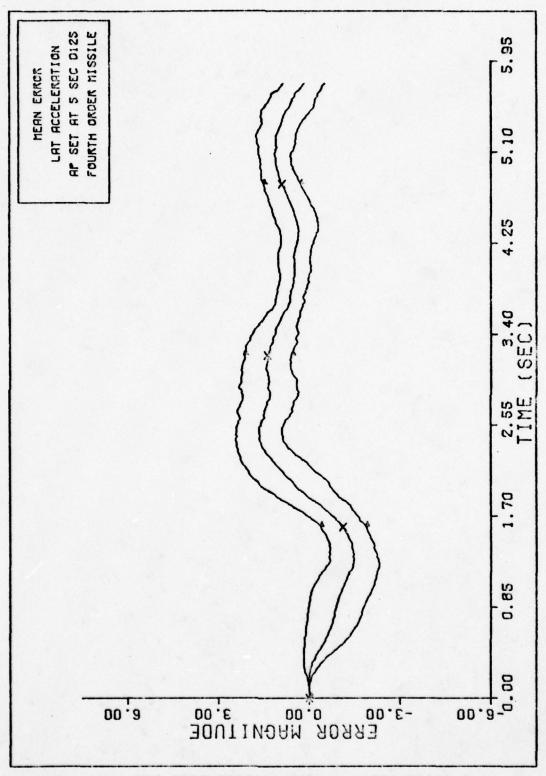


Fig. 60. LAT ACCELERATION FOURTH ORDER MISSILE

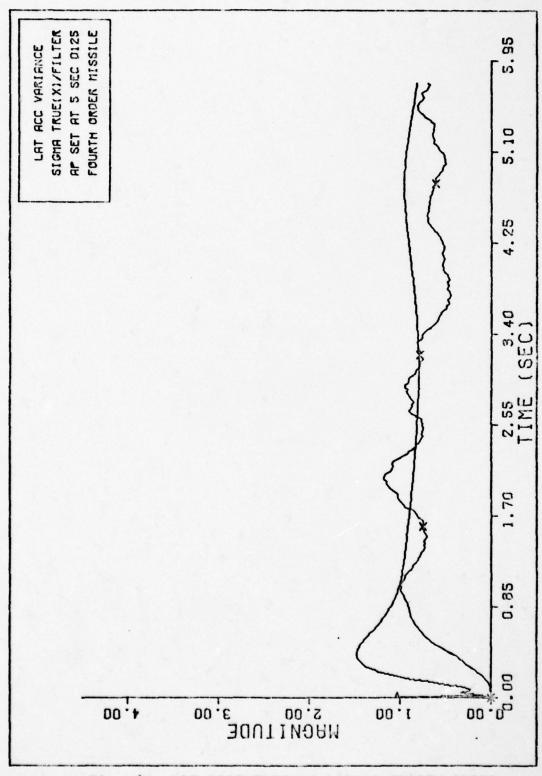


Fig. 61. LAT ACCELERATION SIGMAS FOURTH GROER

## Fourth Order Missile Filter (complete linearization of f)

The initial state estimates and the tuning parameters for this case are

$$V_{mx}^{I}(0) = 1225.7 \text{ fps}$$
 $\dot{\mathbf{e}}(0) = 4.363345 \text{ radians}$ 
 $R(0) = 10000. \text{ feet}$ 
 $\dot{R}(0) = -2122. \text{ fps}$ 
 $a_{L}(0) = 0.$ 
 $n(0) = 4.5$ 
 $\tau_{f}(0) = N/A$ 
 $M/S(0) = 29.197 \text{ slugs/ft}^{2}$ 

$$\underline{\mathbf{R}} = \begin{bmatrix} 4.E-5 & 0. & 0. \\ 0. & 500. & 0. \\ 0. & 0. & 100. \end{bmatrix}$$

These plots were generated to justify the assumptions, that  $\dot{\theta}_T$ ,  $V_m$ , and  $\gamma_m$  could be considered constant over the measurement period of 0.02 seconds. The fourth order filter was used with the coefficients of the autopilot chosen for t=0. The <u>F</u>-matrix within the filter contains a complete linearization of the f-vector. A comparison of these plots with those for "Fourth Order Missile Filter, A/P at t=0" (Figures 32 through 41), can be made to determine the difference when the above mentioned variables are considered constant over the measurement period.

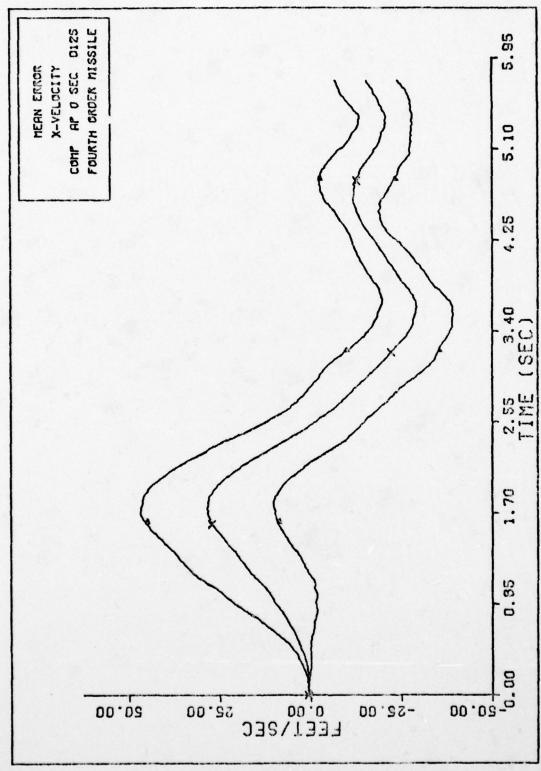


Fig. 62. X-VELOCITY FOURTH ORDER MISSILE

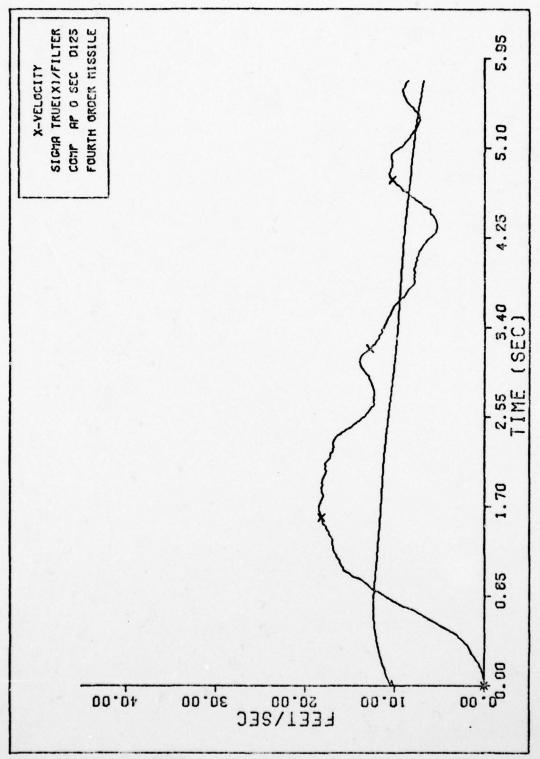


Fig. 63. X-VELOCITY SIGMAS FOURTH ORDER

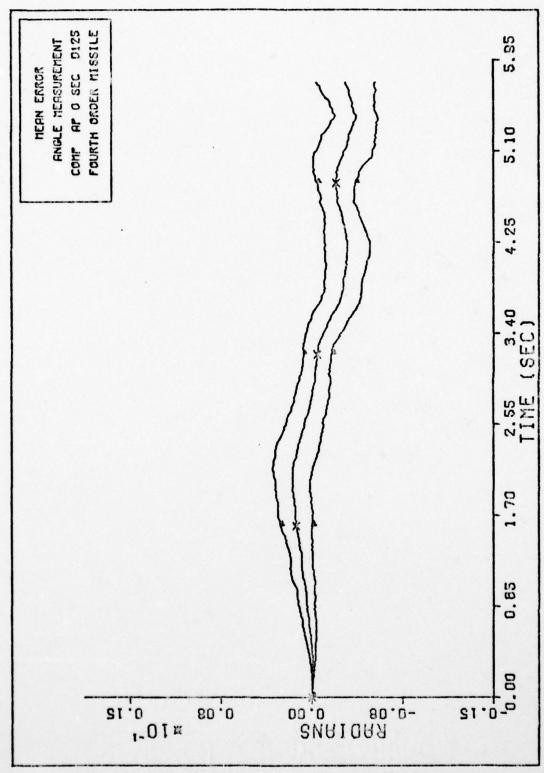


Fig. 64. ANGLE MEASUREMENT FOURTH ORDER MISSILE

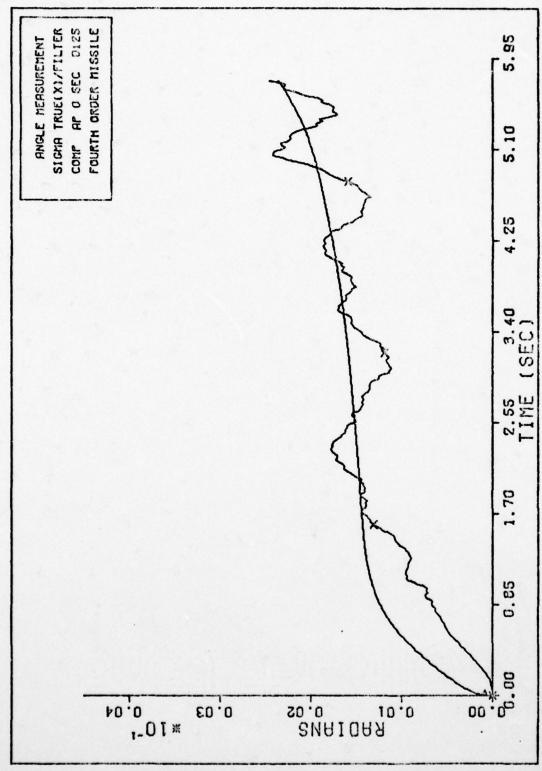


Fig. 65. ANGLE MEASUREMENT SIGMAS FOURTH ORDER

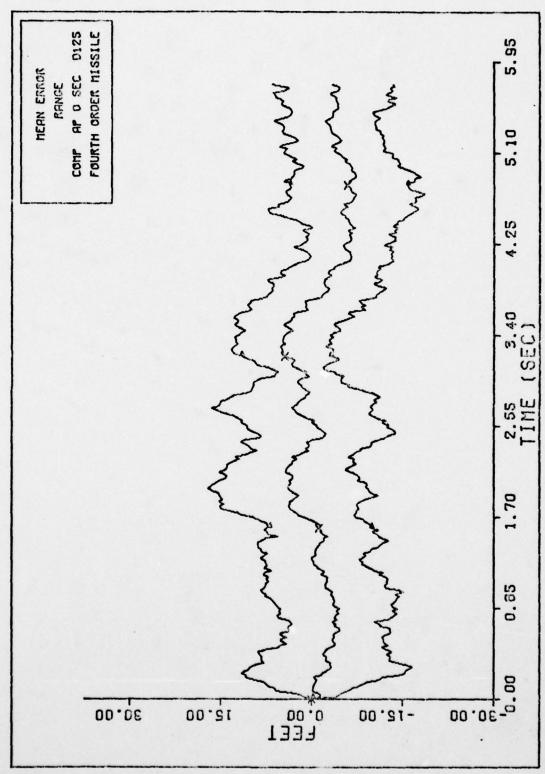


Fig. 66. RANGE FOURTH ORDER MISSILE

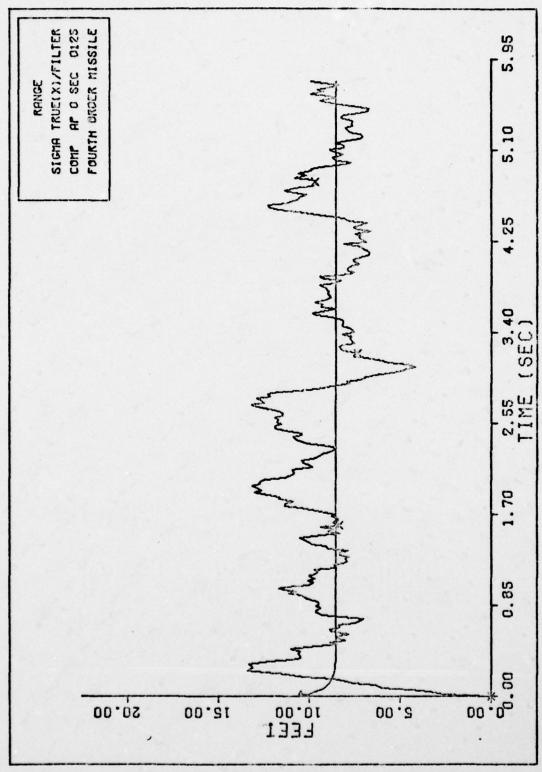


Fig. 67. RANGE SIGMAS FOURTH ORDER

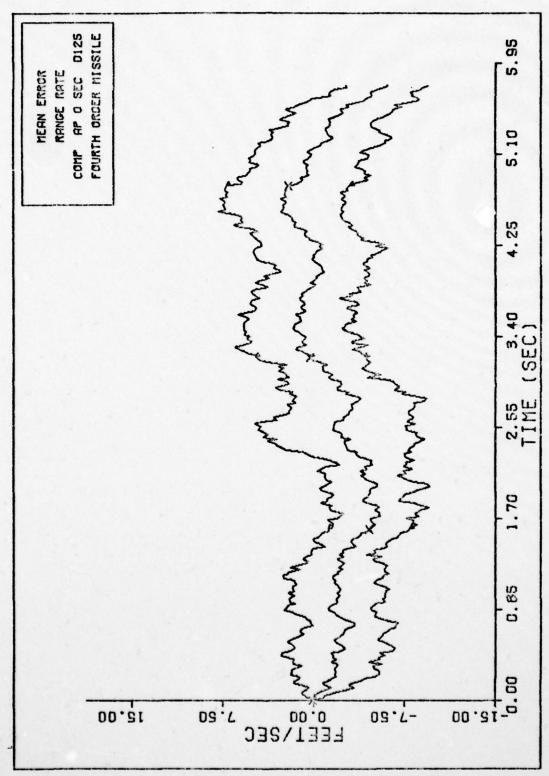


Fig. 68. RANGE RATE FOURTH ORDER MISSILE

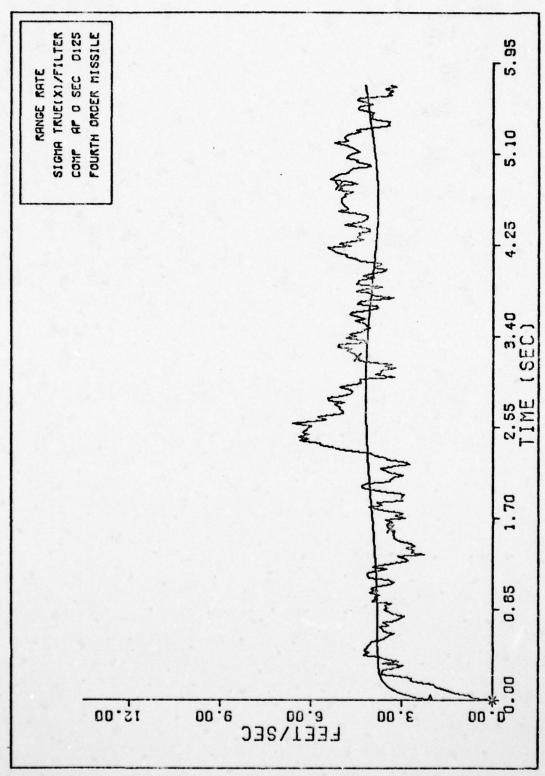


Fig. 69. RANGE RATE SIGMAS FOURTH GROER

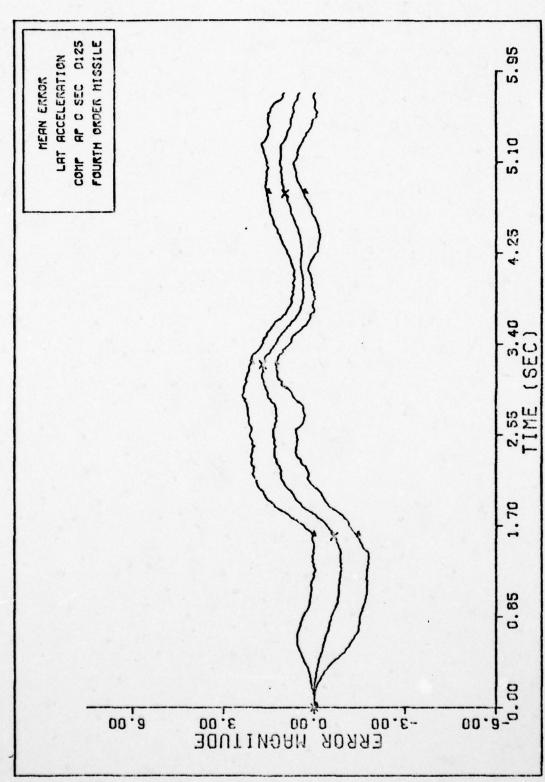


Fig. 70. LAT ACCELERATION FOURTH ORDER MISSILE

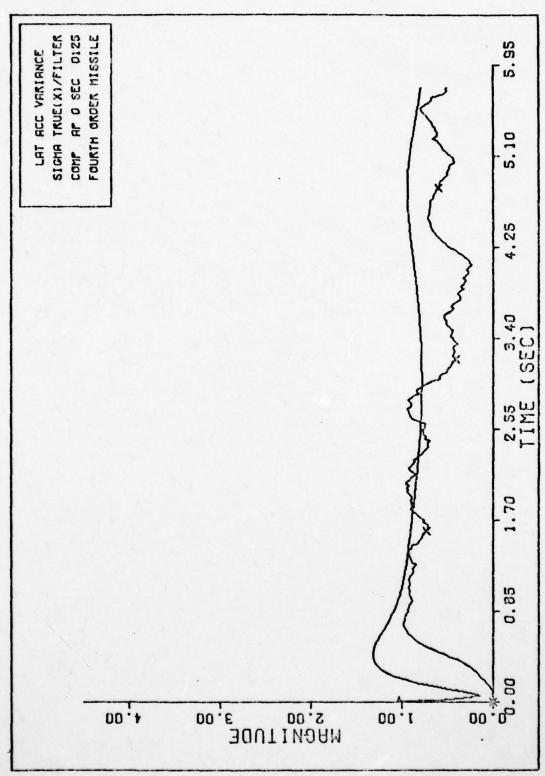


Fig. 71. LAT ACCELERATION SIGNAS FOURTH ORDER

Fourth Order Missile Filter (using Runge-Kutta integration)

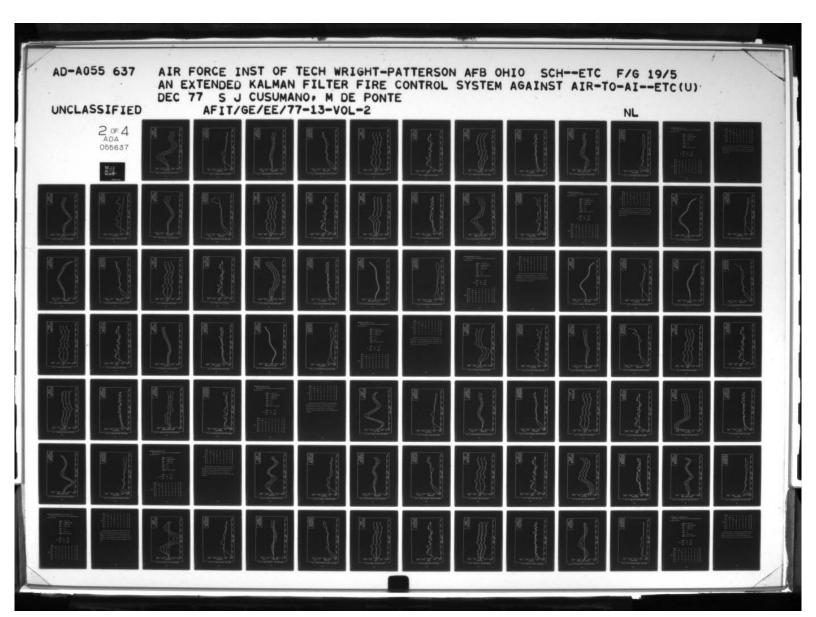
The initial state estimates and the tuning parameters

for this case are

$$V_{mx}^{I}(0) = 1225.7 \text{ fps}$$
 $\dot{\theta}(0) = 4.363345 \text{ radians}$ 
 $R(0) = 10000. \text{ feet}$ 
 $\dot{R}(0) = -2122. \text{ fps}$ 
 $a_{L}(0) = 0.$ 
 $n(0) = 4.5$ 
 $T_{f}(0) = N/A$ 
 $M/S(0) = 29.197 \text{ slugs/ft}^{2}$ 

$$\underline{\mathbf{R}} = \begin{bmatrix} 4.E-5 & 0. & 0. \\ 0. & 500. & 0. \\ 0. & 0. & 100. \end{bmatrix}$$

These plots were generated to determine if the filter performance was degraded by using a first order Euler integration routine. The results shown here used a Runge-Kutta routine and can be compared to Figures 52 through 61, which were generated with the Euler routine. The coefficients of the autopilot were chosen for t=5, since these coefficients caused the greatest transients in the dynamic states of the filter (compared to t=0 and t=3, as described in Chapter III). It was decided that a favorable comparison for this case would be the best test for the Euler routine.



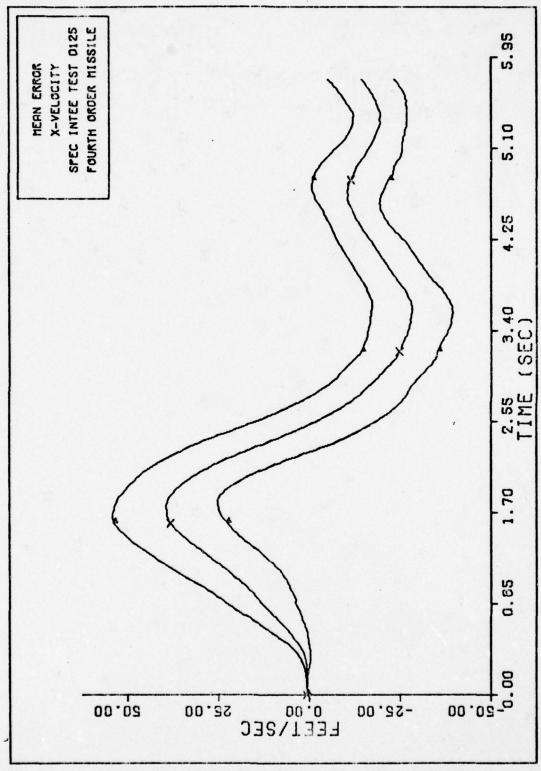


Fig. 72. X-VELOCITY FOURTH ORDER MISSILE

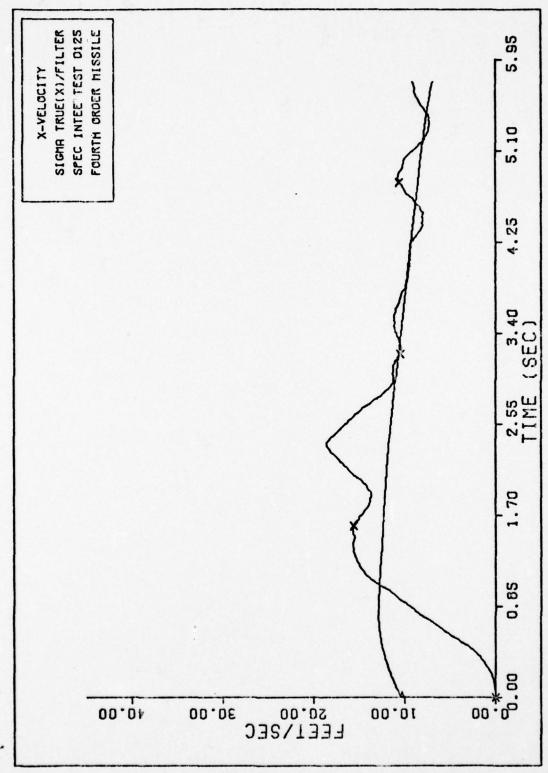


Fig. 73. X-VELOCITY SIGMAS FOURTH ORDER

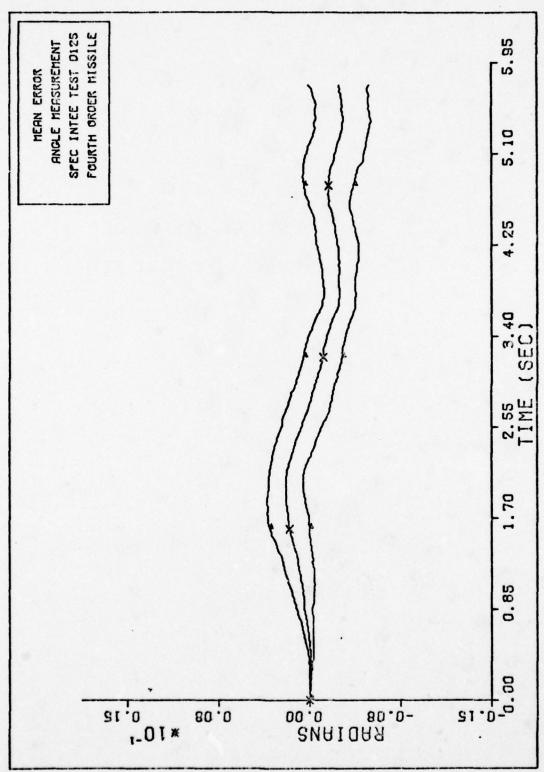


Fig. 74. ANGLE MEASUREMENT FOURTH ORDER MISSILE

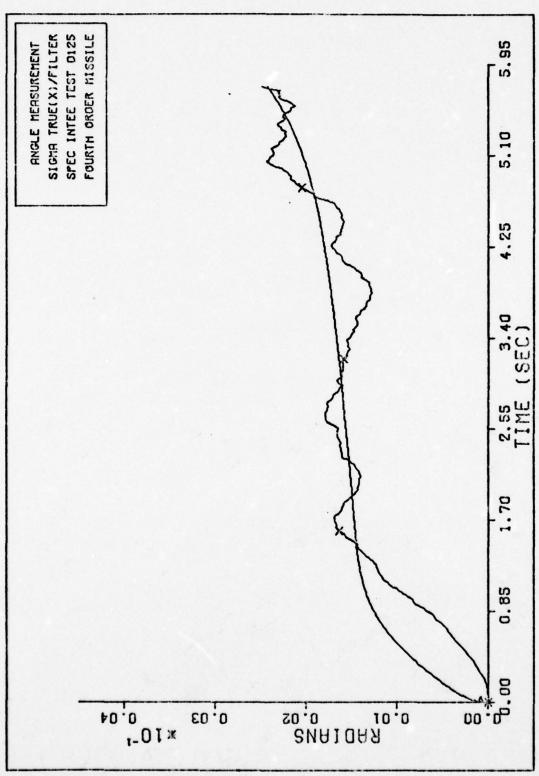


Fig. 75. ANGLE MEASUREMENT SIGMAS FOURTH ORDER

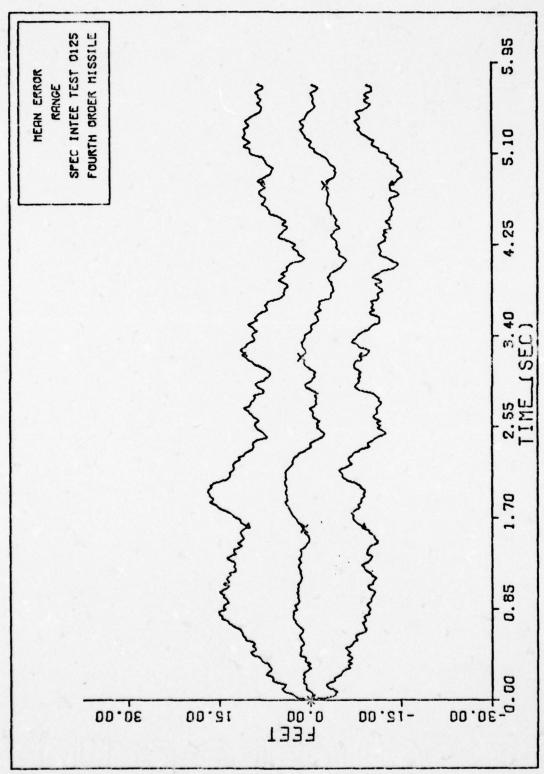


Fig. 76. RANGE FOURTH ORDER MISSILE

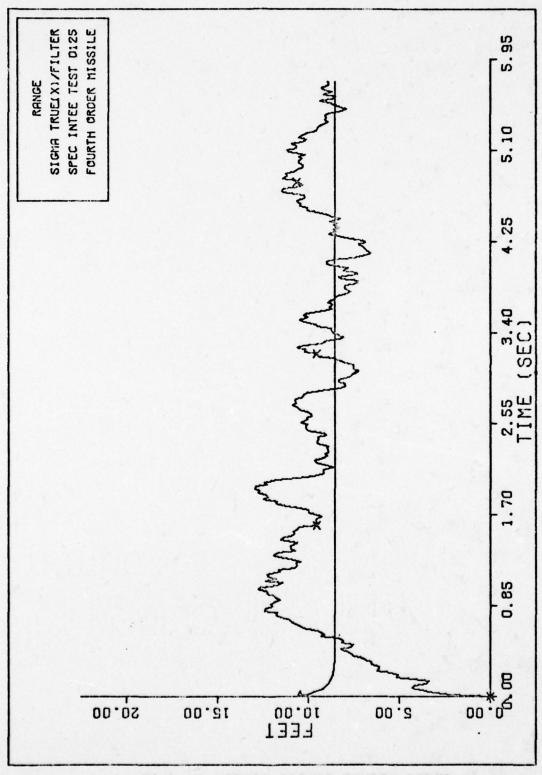


Fig. 77. RANGE SIGMAS FOURTH ORDER

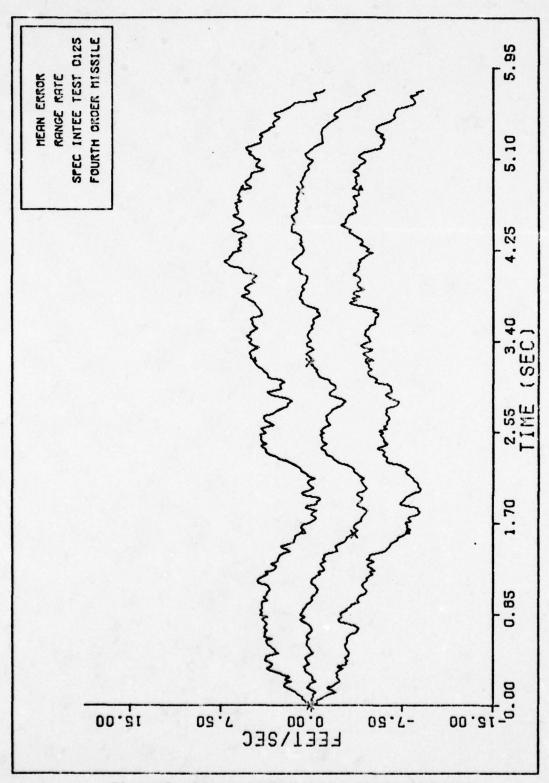


Fig. 78. RANGE RATE FOURTH ORDER MISSILE

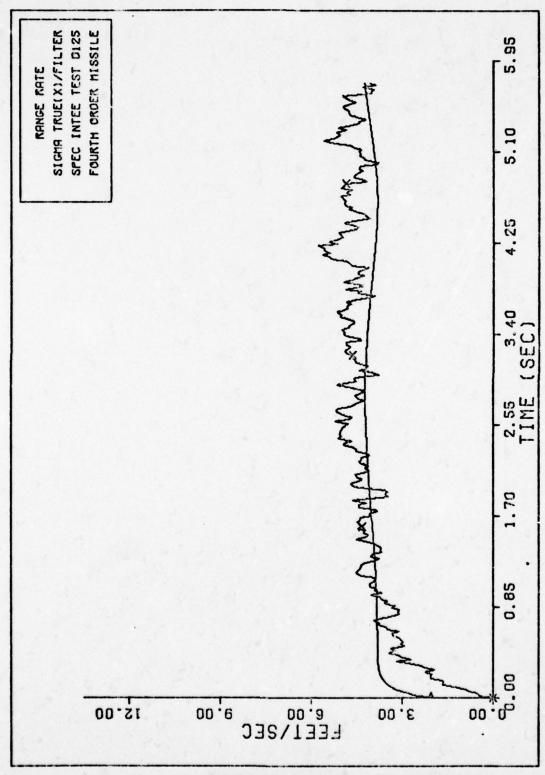


Fig. 79. RANGE RATE SIGMAS FOURTH ORDER

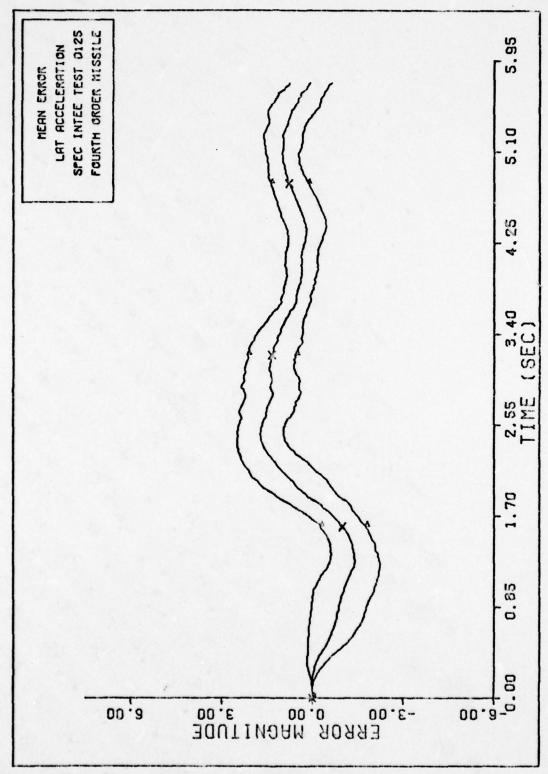


Fig. 80. LAT ACCELERATION FOURTH ORDER MISSILE

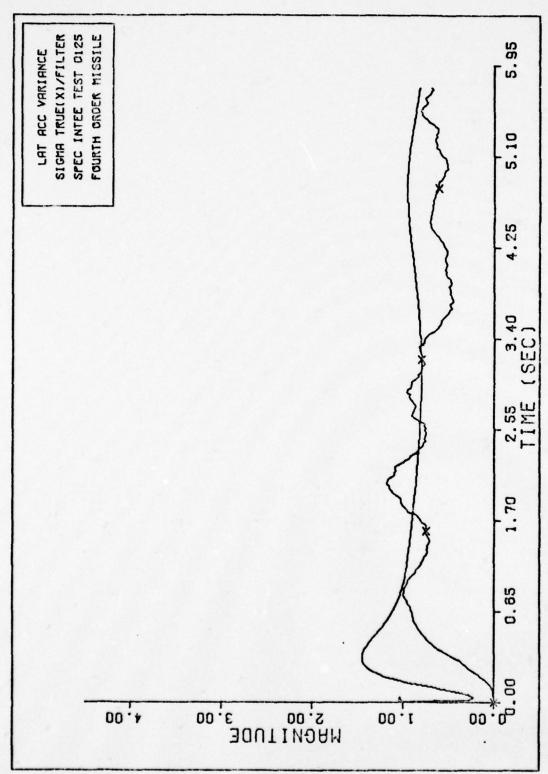


Fig. 81. LAT ACCELERATION SIGMAS FOURTH ORDER

## Sensitivity Analysis (n = 6.)

The initial state estimates and the tuning parameters for this case are

$$V_{mx}^{I}(0) = 1225.7 \text{ fps}$$
 $\dot{q}(0) = 4.363345 \text{ radians}$ 
 $R(0) = 10000. \text{ feet}$ 
 $\dot{R}(0) = -2122. \text{ fps}$ 
 $a_{L}(0) = 0.$ 
 $n(0) = 4.5$ 
 $\tau_{f}(0) = N/A$ 
 $M/S(0) = 29.197 \text{ slugs/ft}^{2}$ 

$$\underline{\mathbf{R}} = \begin{bmatrix} \mathbf{T} \cdot \mathbf{E} - 5 & 0 & 0 & 0 \\ 0 & 500 & 0 & 0 \\ 0 & 0 & 100 & 0 \end{bmatrix}$$

This set of plots was generated by setting the proportional navigation constant, n, to 6 in the truth model. The fourth order filter was used with n set to 4.5 in the filter. Only the dynamic states of the missile model were estimated by the filter.

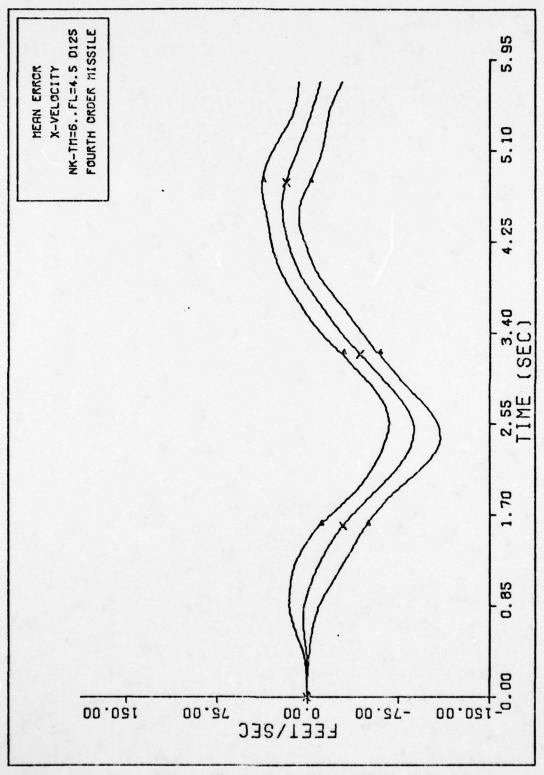


Fig. 82. X-VELOCITY FOURTH ORDER MISSILE

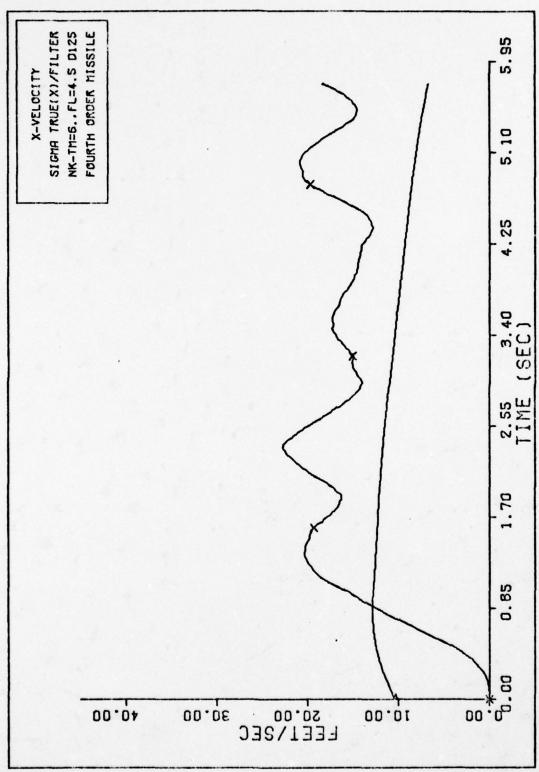


Fig. 83. X-VELOCITY SIGMAS FOURTH ORDER

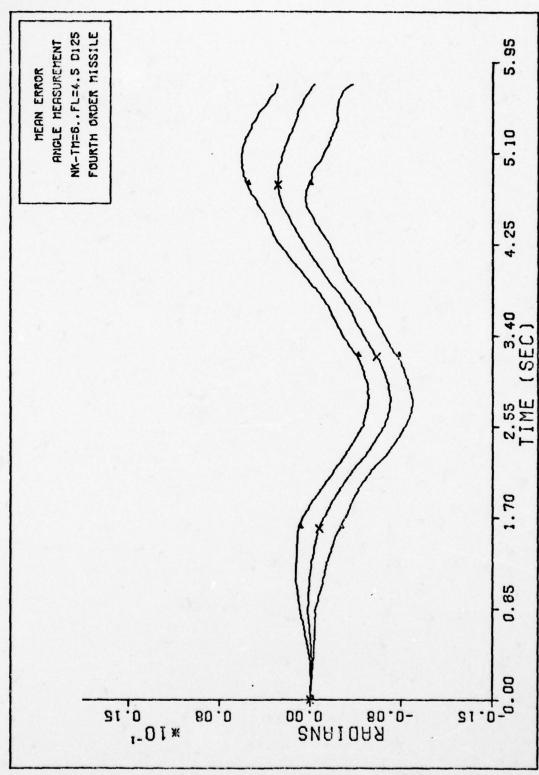


Fig. 84. ANGLE MEASUREMENT FOURTH ORDER MISSILE

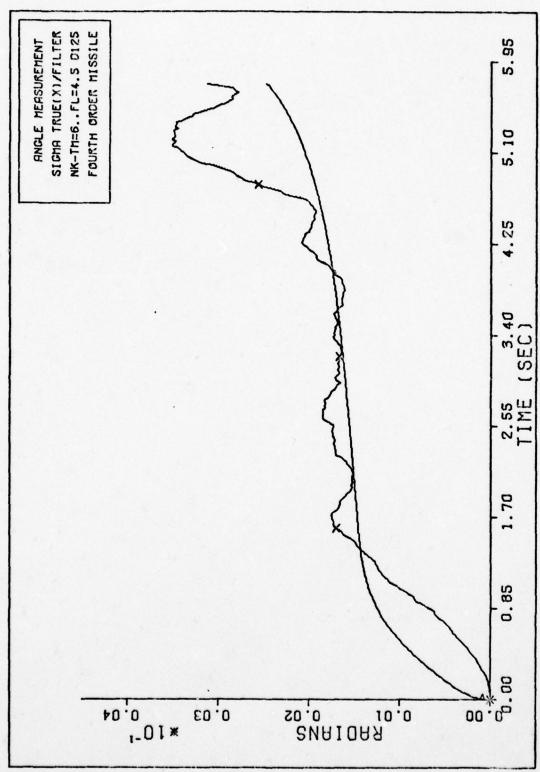


Fig. 85. ANGLE MEASUREMENT SIGMAS FOURTH ORDER

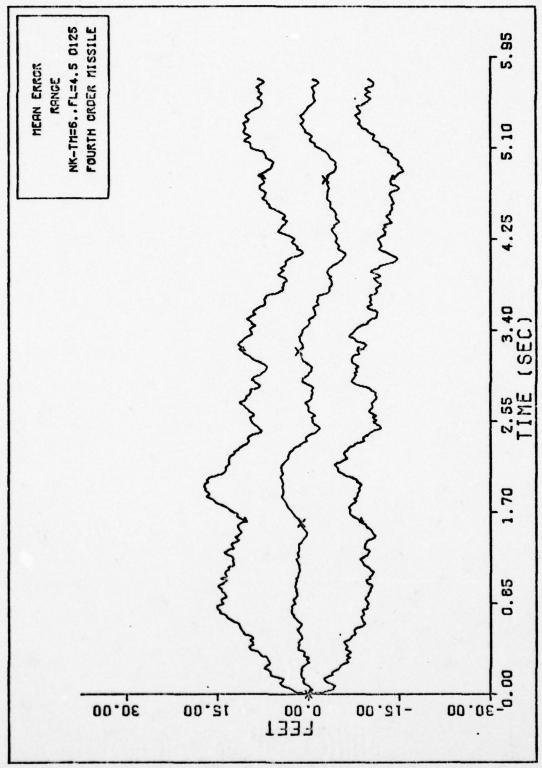


Fig. 86. RANGE FOURTH ORDER MISSILE

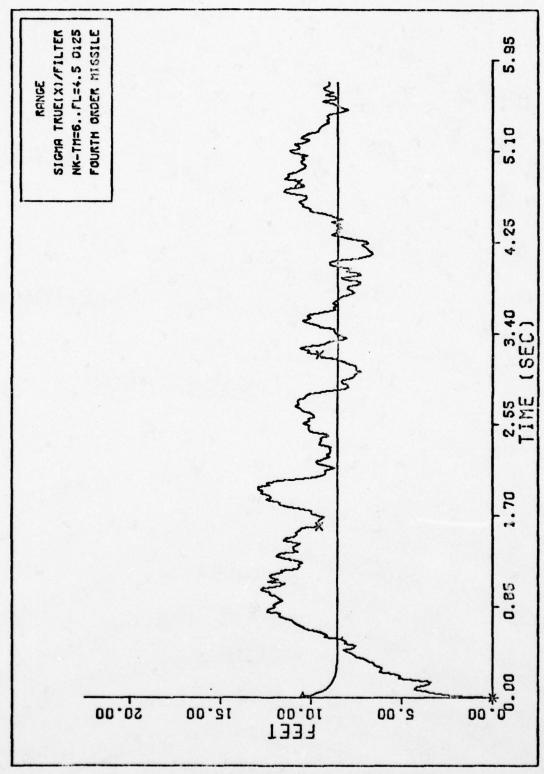


Fig. 87. RANGE SIGMAS FOURTH ORDER

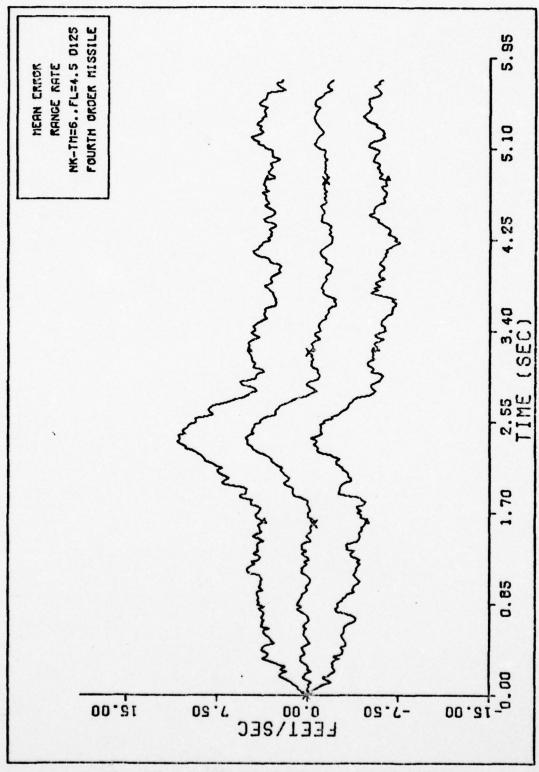


Fig. 88. RANGE RATE FOURTH CROER MISSILE

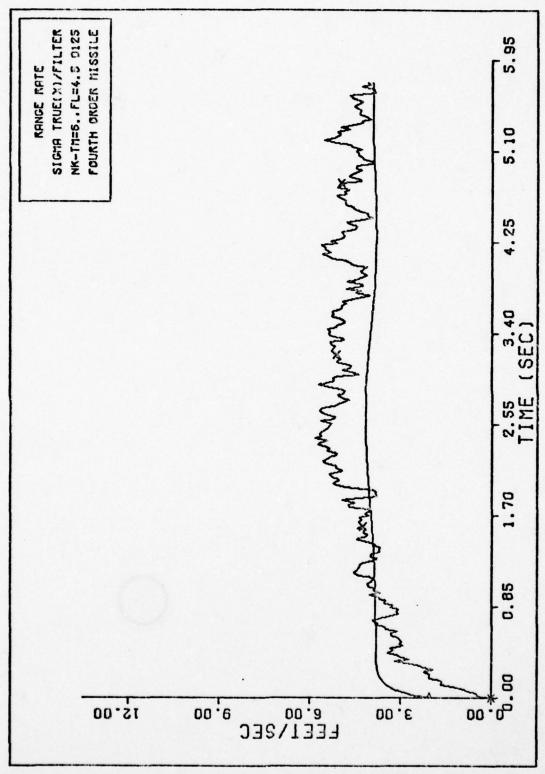


Fig. 89. RANGE RATE SIGMAS FOURTH ORDER

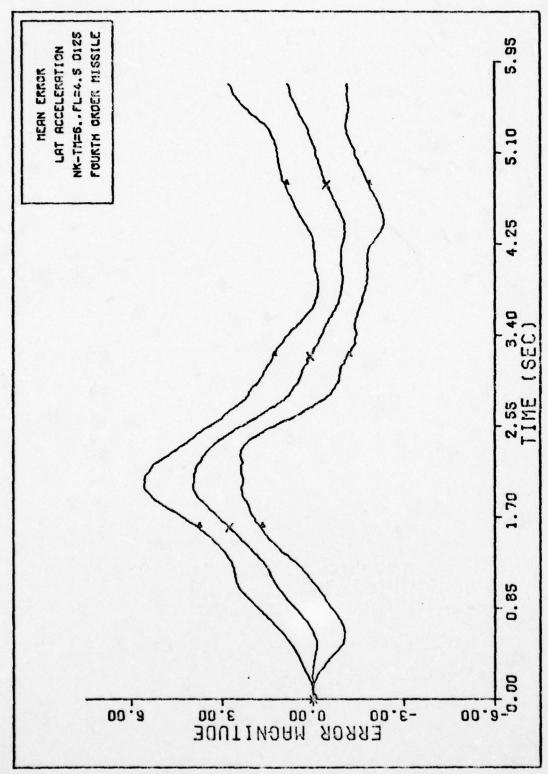


Fig. 90. LAT ACCELERATION FOURTH ORDER MISSILE

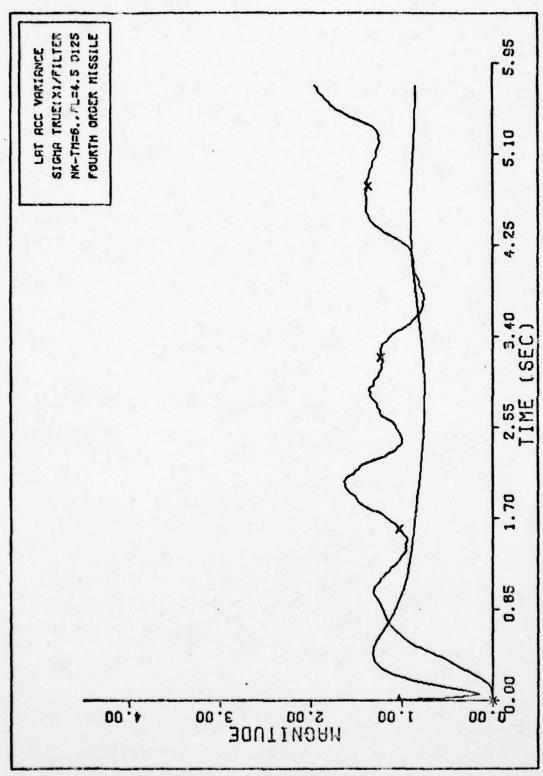


Fig. 91. LAT ACCELERATION SIGMAS FOURTH DRDER

## Sensitivity Analysis (n = 3.)

The initial state estimates and the tuning parameters for this case are

$$v_{mx}^{I}(0) = 1225.7 \text{ fps}$$
 $e(0) = 4.363345 \text{ radians}$ 
 $R(0) = 10000. \text{ feet}$ 
 $\dot{R}(0) = -2122. \text{ fps}$ 
 $a_{L}(0) = 0.$ 
 $n(0) = 4.5$ 
 $\tau_{f}(0) = N/A$ 
 $M/S(0) = 29.197 \text{ slugs/ft}^{2}$ 

$$\underline{\mathbf{R}} = \begin{bmatrix} \overline{\mathbf{4}} \cdot \mathbf{E} - 5 & 0 & 0 \\ 0 & 500 & 0 \\ 0 & 0 & 100 \end{bmatrix}$$

This set of plots was generated by setting the proportional navigation constant, n, to 3 in the truth model. The fourth order filter was used with n set to 4.5 in the filter. Only the dynamic states of the missile model were estimated by the filter.

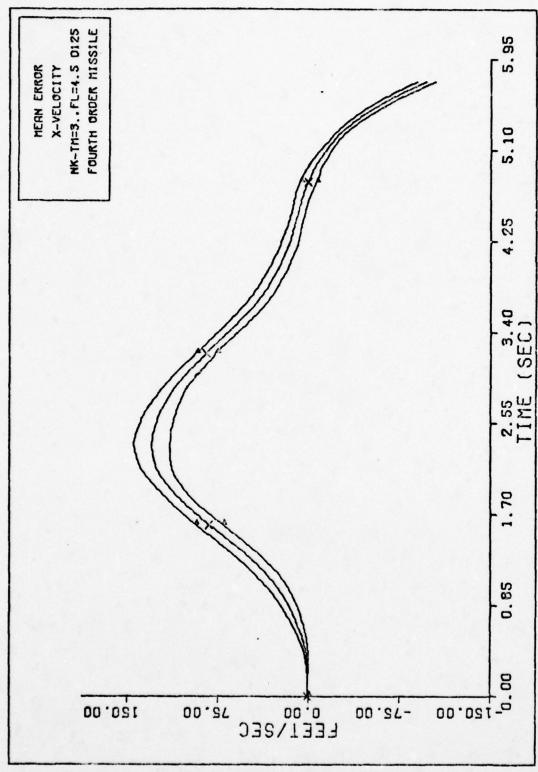


Fig. 92. X-VELOCITY FOURTH ORDER MISSILE

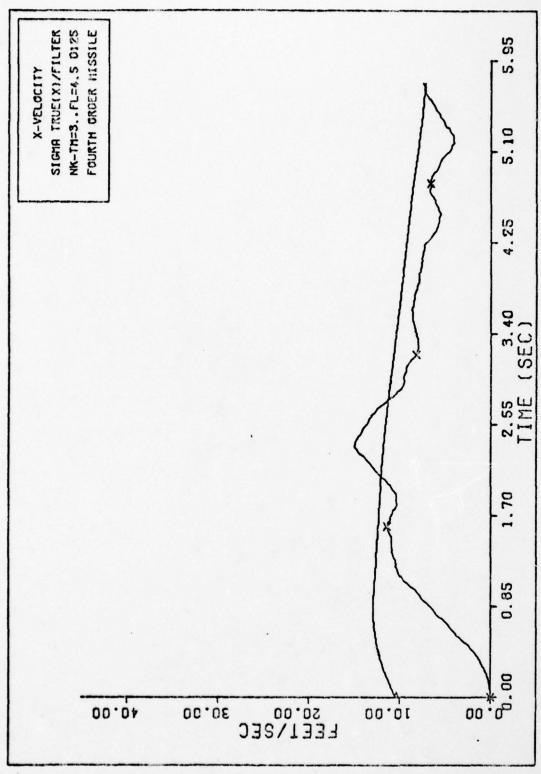


Fig. 93. X-VELOCITY SIGMAS FOURTH GROER

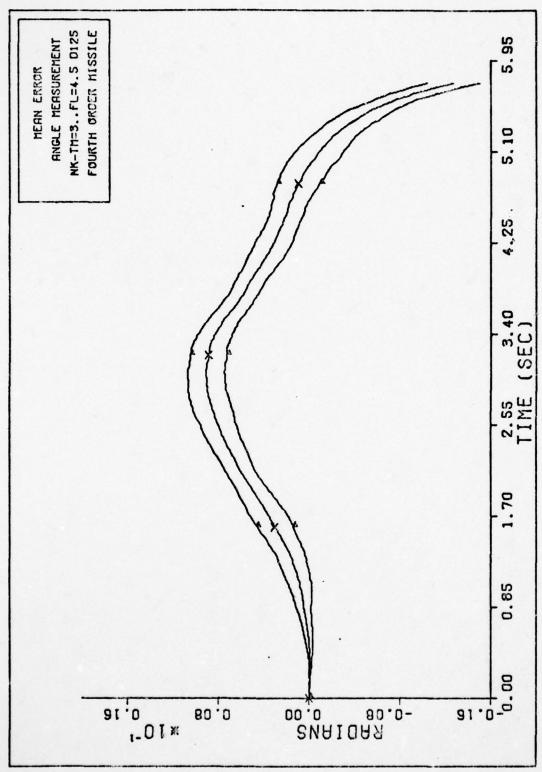


Fig. 94. ANGLE MEASUREMENT FOURTH ORDER MISSILE

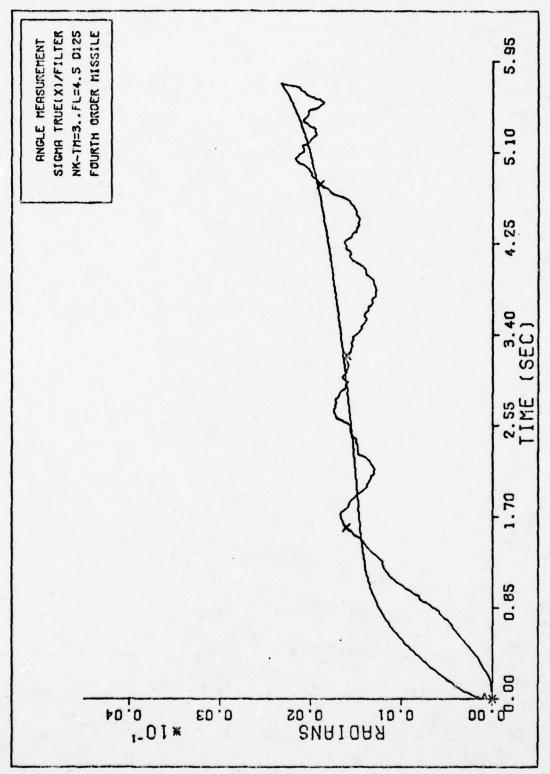


Fig. 95. ANGLE MEASUREMENT SIGMAS FOURTH ORDER

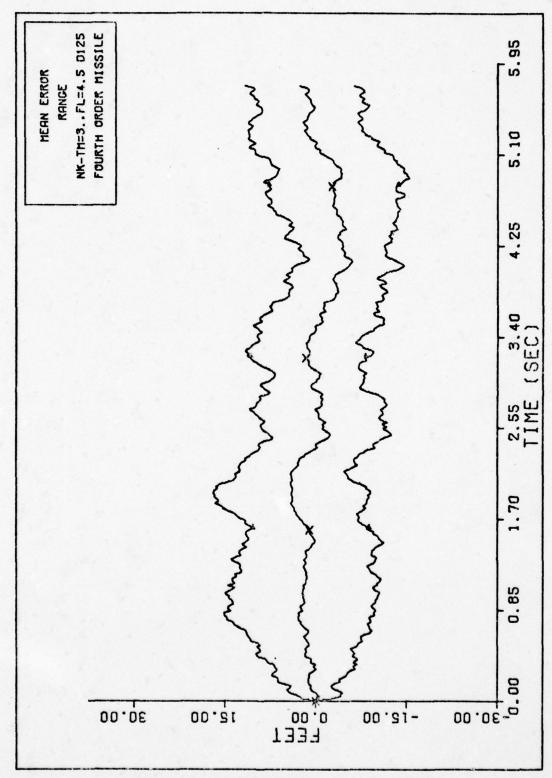


Fig. 96. RANGE FOURTH ORDER MISSILE

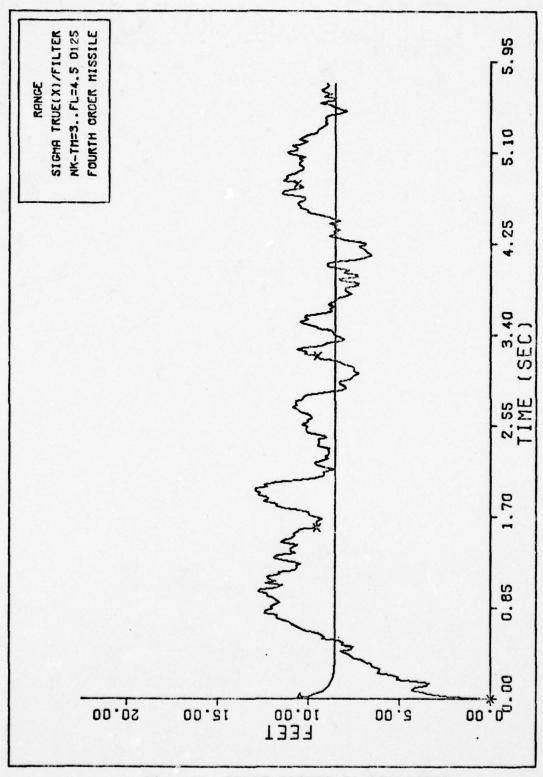


Fig. 97. RANGE SIGMAS FOURTH ORDER

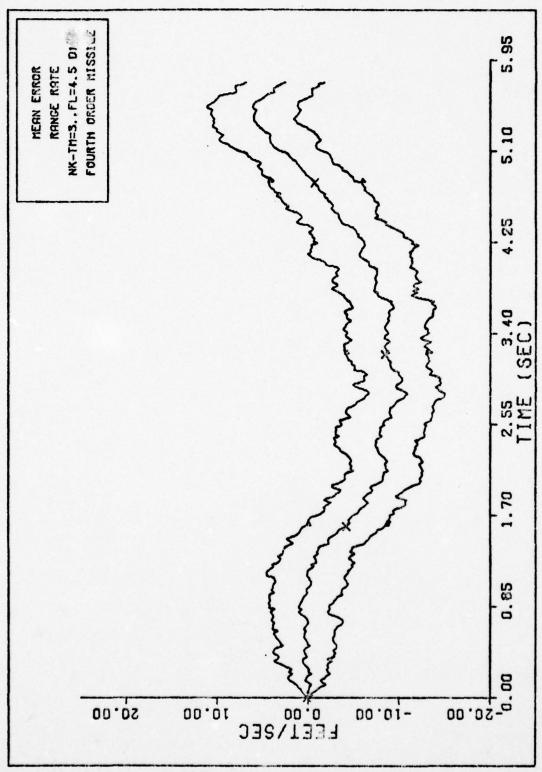


Fig. 98. RANGE RATE FOURTH ORDER MISSILE

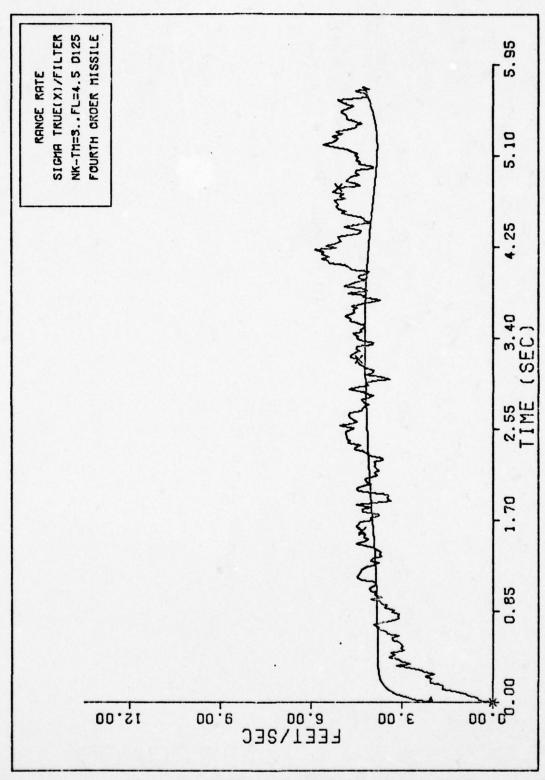


Fig. 99. RANGE RATE SIGMAS FOURTH ORDER

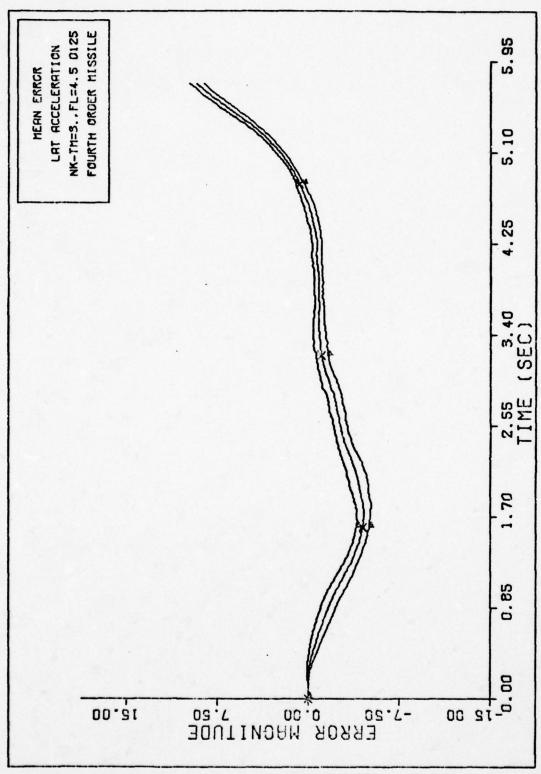


Fig. 100. LAT ACCELERATION FOURTH ORDER MISSILE

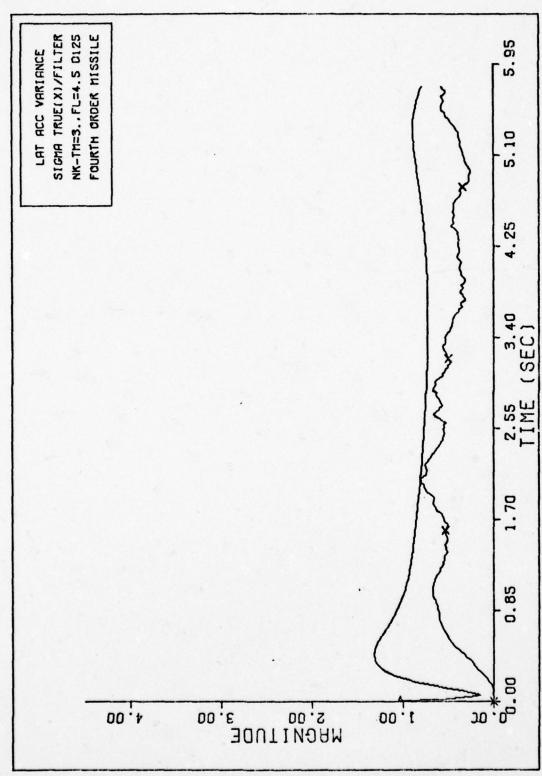


Fig. 101. LAT ACCELERATION SIGMAS FOURTH ORDER

## Sensitivity Analysis $(\underline{\tau}_2 = .8)$

The initial state estimates and the tuning parameters for this case are

$$V_{mx}^{I}(0) = 1225.7 \text{ fps}$$
  
 $\theta(0) = 4.363345 \text{ radians}$   
 $R(0) = 10000. \text{ feet}$   
 $\dot{R}(0) = -2122. \text{ fps}$   
 $a_{L}(0) = 0.$   
 $n(0) = 4.5$   
 $\tau_{f}(0) = N/A$   
 $M/S(0) = 29.197 \text{ slugs/ft}^{2}$ 

This set of plots was generated by setting the time constant of the guidance system,  $\tau_2$  to .8 seconds in the truth model.  $\tau_2$  in the filter was set to 0.3 seconds. The fourth order filter was used and only the dynamic states of the missile model were estimated.

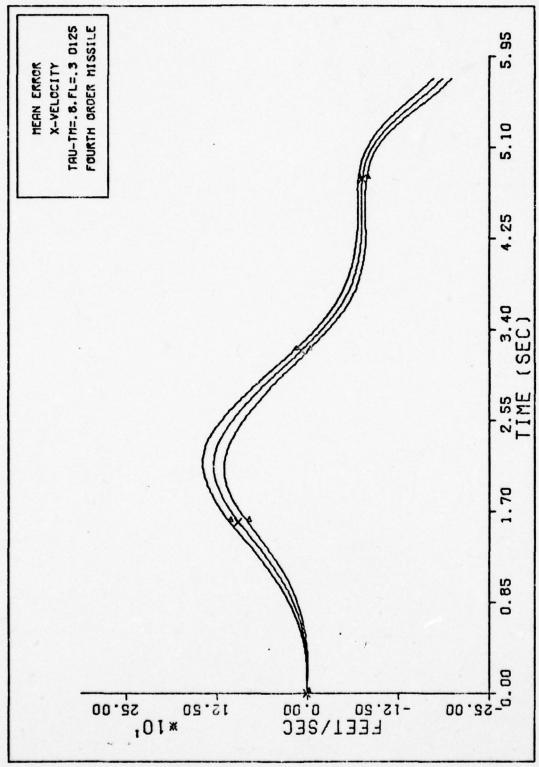


Fig. 102. X-VELOCITY FOURTH ORDER MISSILE

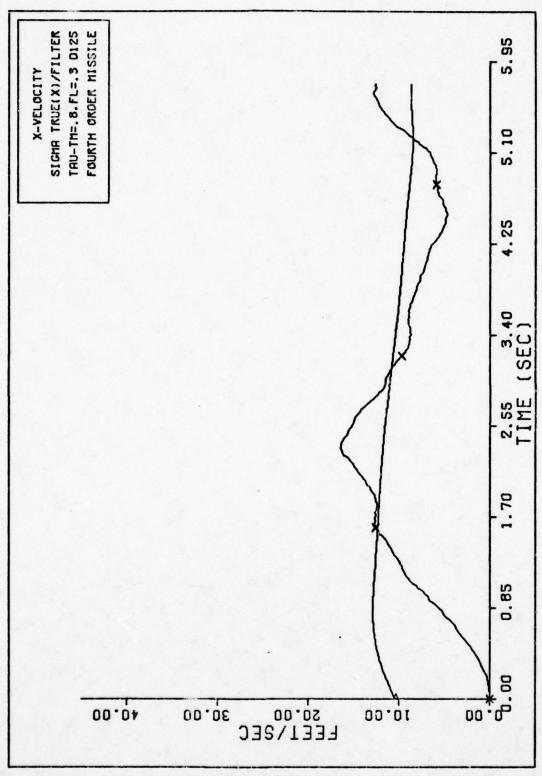


Fig. 103. X-VELOCITY SIGMAS FOURTH ORDER

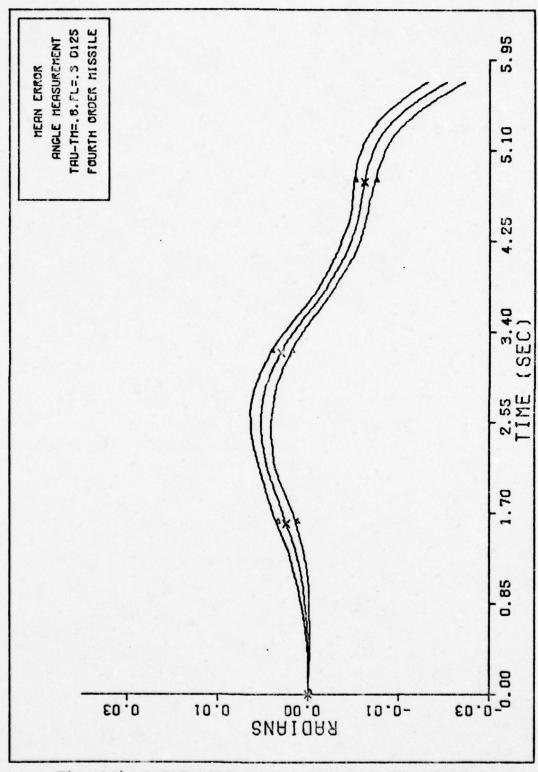


Fig. 104. ANGLE MEASUREMENT FOURTH ORDER MISSILE

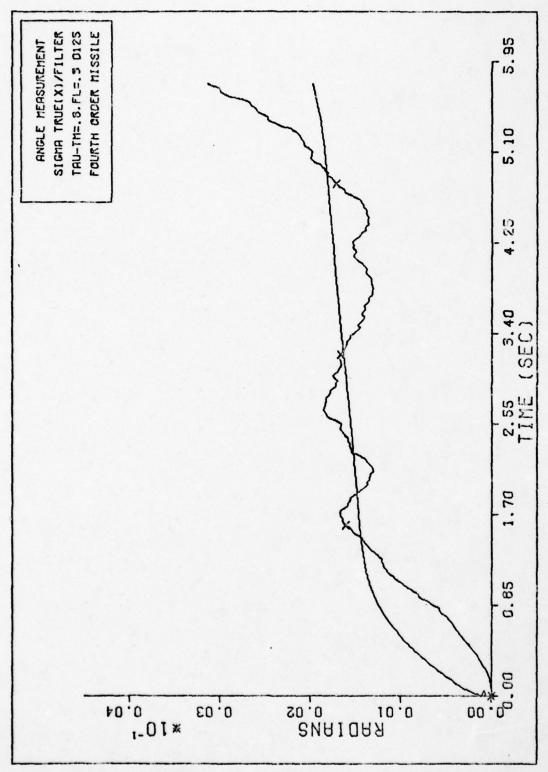


Fig. 105. ANGLE MERSUREMENT SIGMAS FOURTH ORDER

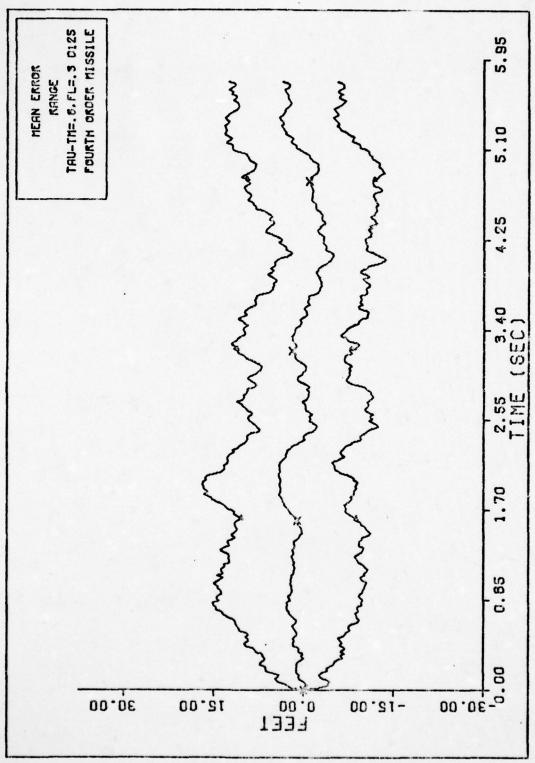


Fig. 106. RANGE FOURTH ORDER MISSILE

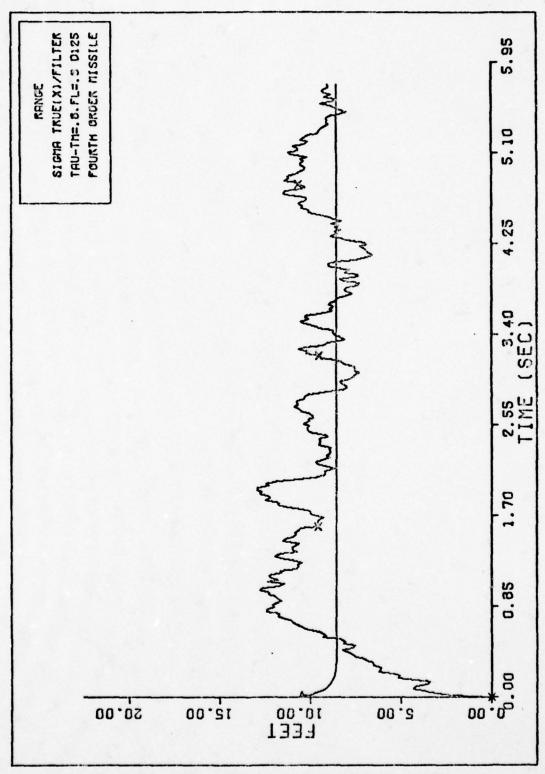


Fig. 107. RANGE SIGMAS FOURTH ORDER

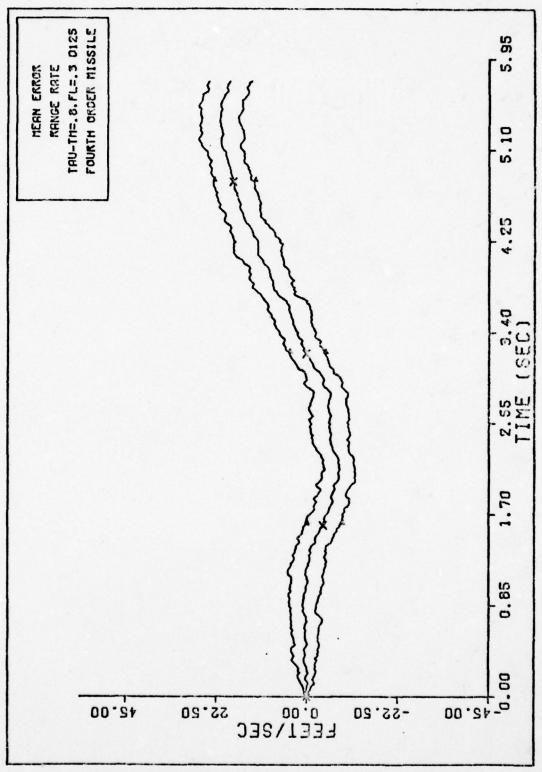


Fig. 108. RANGE RATE FOURTH ORDER MISSILE

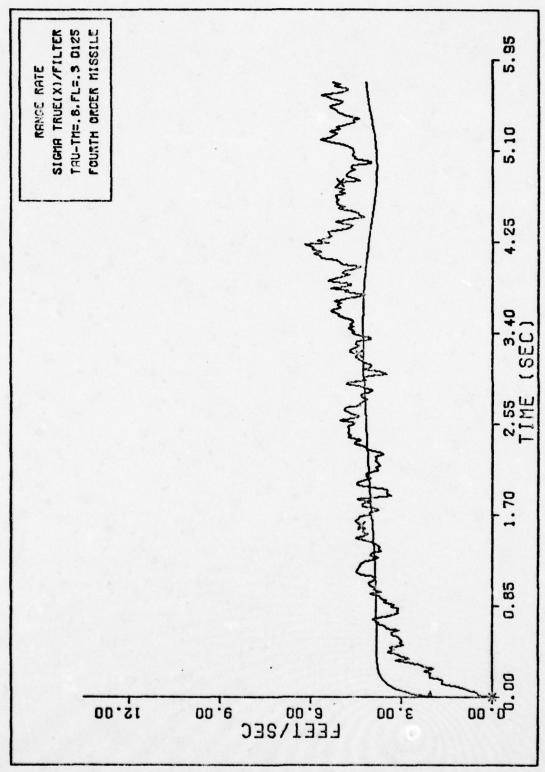


Fig. 109. RANGE RATE SIGMAS FOURTH ORDER

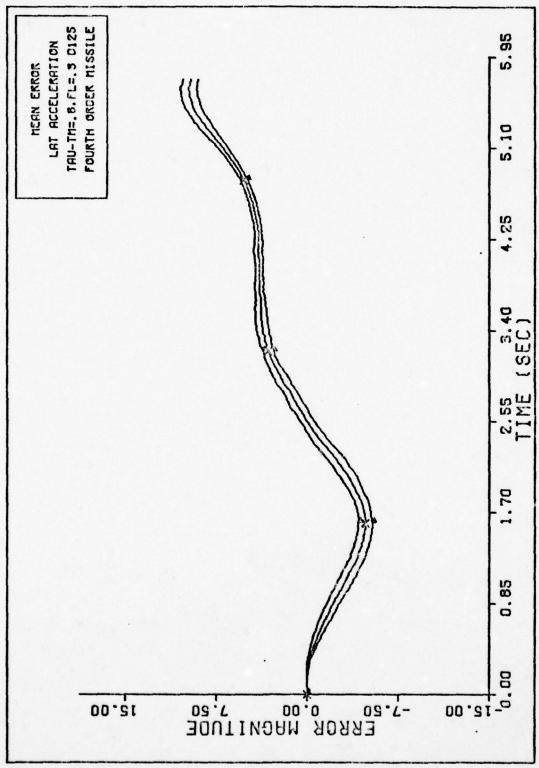


Fig. 110. LAT ACCELERATION FOURTH ORDER MISSILE

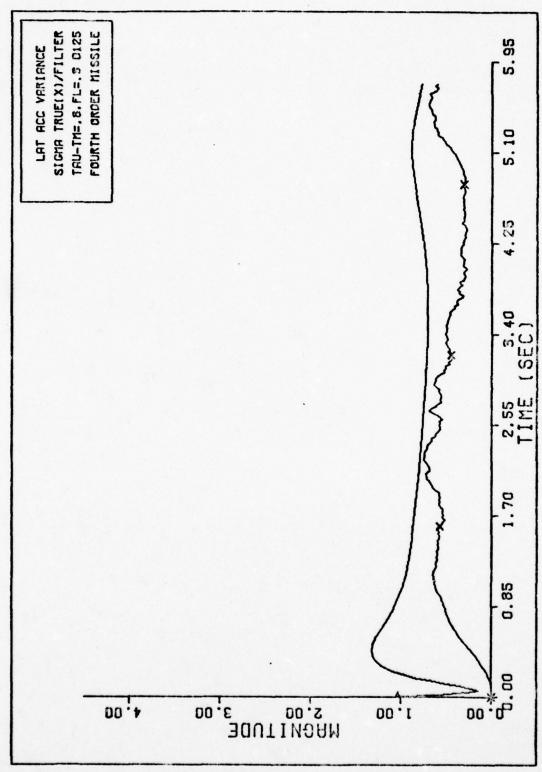


Fig. 111. LAT ACCELERATION SIGNAS FOURTH ORDER

## <u>Sensitivity</u> <u>Analysis</u> $(\underline{\tau}_2 = .1)$

The initial state estimates and the tuning parameters for this case are

$$V_{mx}^{I}(0) = 1225.7 \text{ fps}$$
 $\dot{\theta}_{I}(0) = 4.363345 \text{ radians}$ 
 $R(0) = 10000. \text{ feet}$ 
 $\dot{R}(0) = -2122. \text{ fps}$ 
 $a_{L}(0) = 0.$ 
 $n(0) = 4.5$ 
 $\tau_{f}(0) = N/A$ 
 $M/S(0) = 29.197 \text{ slugs/ft}^{2}$ 

$$\underline{\mathbf{R}} = \begin{bmatrix} 4.E-5 & 0. & 0. \\ 0. & 500. & 0. \\ 0. & 0. & 100. \end{bmatrix}$$

This set of plots was generated by setting the time constant of the guidance system,  $\tau_2$ , to .1 seconds in the truth model.  $\tau_2$  in the filter was set to 0.3 seconds. The fourth order filter was used and only the dynamic states of the missile model were estimated.

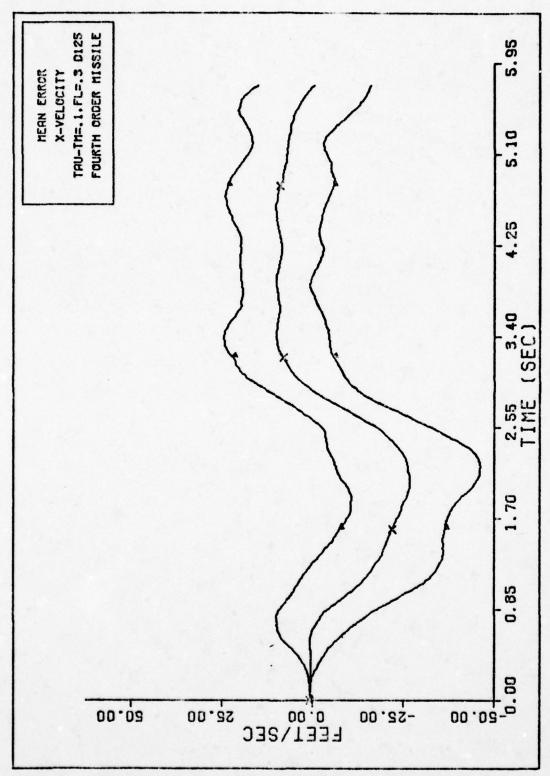


Fig. 112. X-VELOCITY FOURTH ORDER MISSILE

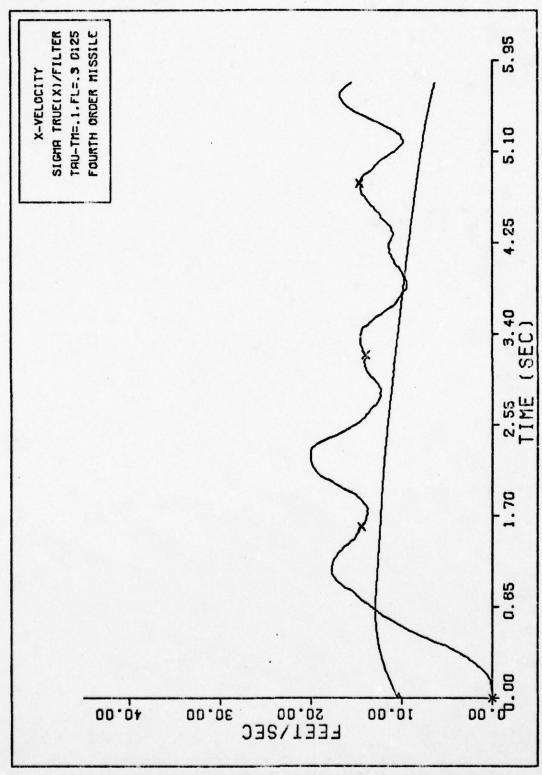


Fig. 113. X-VELOCITY SIGMAS FOURTH ORDER

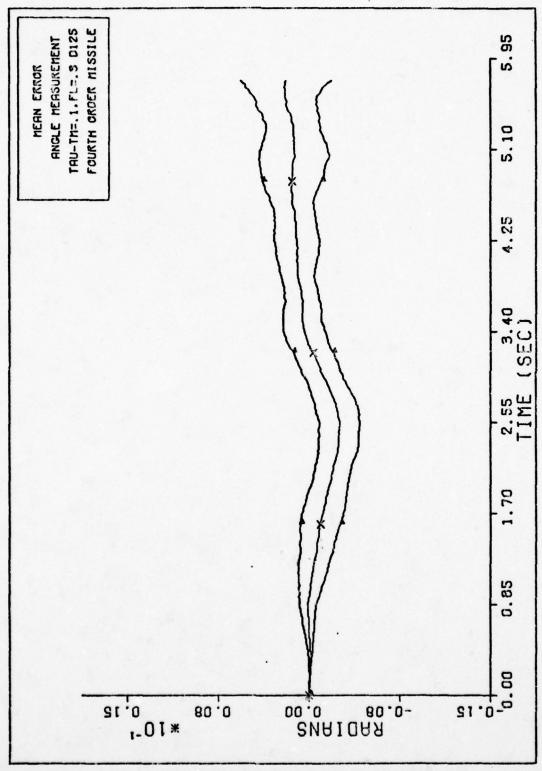


Fig. 114. ANGLE MEASUREMENT FOURTH ORDER MISSILE

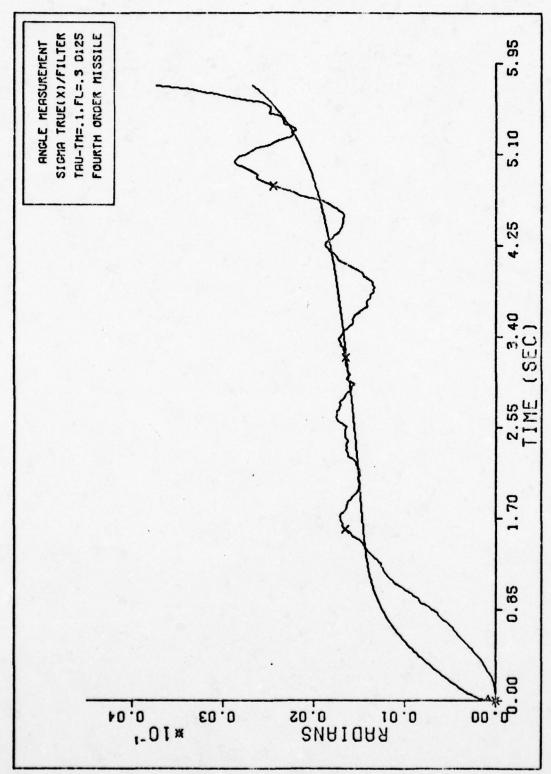


Fig. 115. ANGLE MEASUREMENT SIGNAS FOURTH ORDER

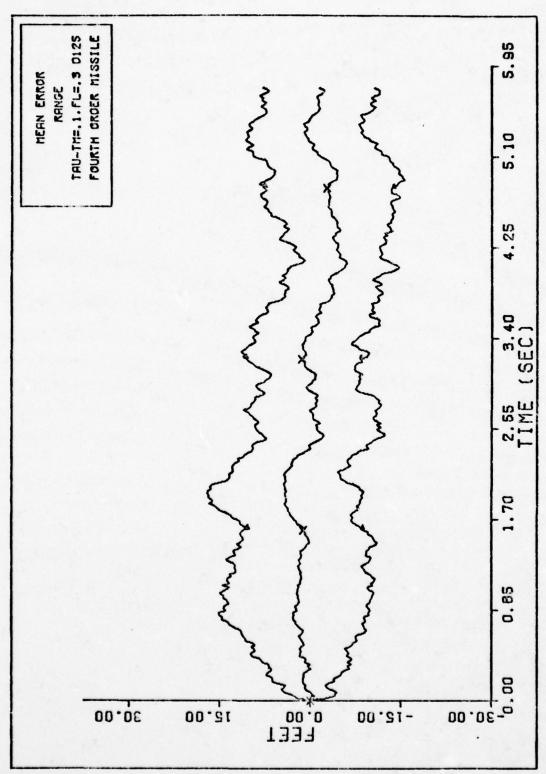


Fig. 116. RANGE FOURTH ORDER MISSILE

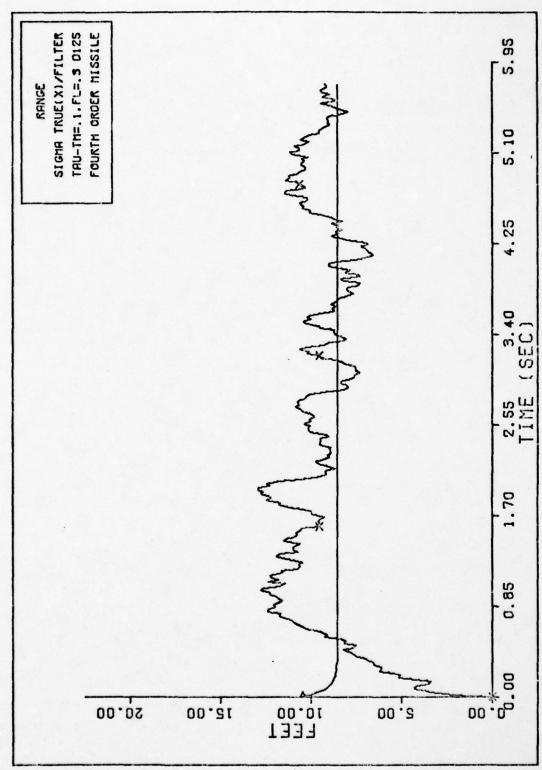


Fig. 117. RANGE SIGMAS FOURTH ORDER

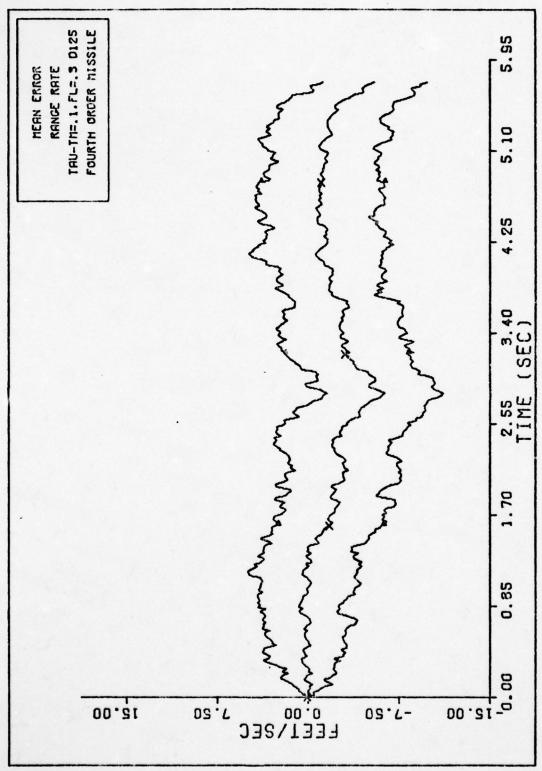


Fig. 118. RANGE RATE FOURTH ORDER MISSILE

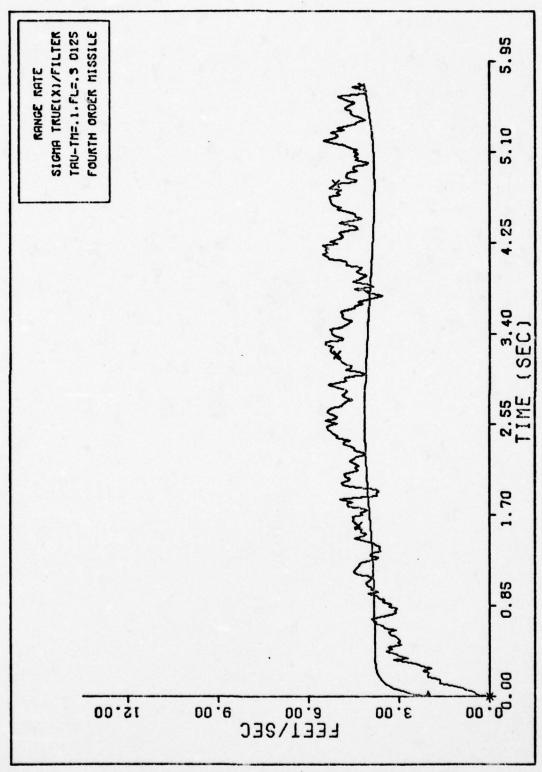


Fig. 119. RANGE RATE SIGMAS FOURTH ORDER

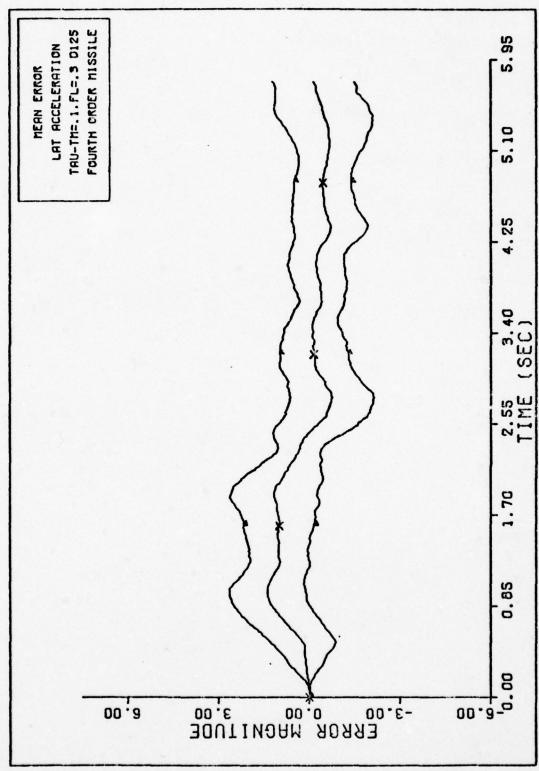


Fig. 120. LAT ACCELERATION FOURTH ORDER MISSILE

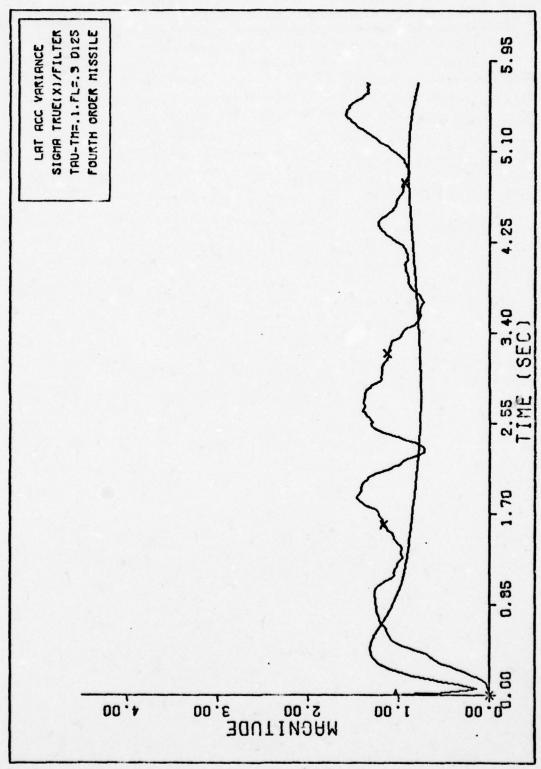


Fig. 121. LAT ACCELERATION SIGMAS FOURTH ORDER

## Sensitivity Analysis (M = 2.)

The initial state estimates and the tuning parameters for this case are

$$V_{mx}^{I}(0) = 1225.7 \text{ fps}$$
 $\dot{\mathbf{e}}_{(0)} = 4.363345 \text{ radians}$ 
 $R(0) = 10000. \text{ feet}$ 
 $\dot{\mathbf{R}}(0) = -2122. \text{ fps}$ 
 $\mathbf{a}_{L}(0) = 0.$ 
 $\mathbf{n}(0) = 4.5$ 
 $\mathbf{\tau}_{f}(0) = N/A$ 
 $M/S(0) = 29.197 \text{ slugs/ft}^{2}$ 

$$\underline{\mathbf{R}} = \begin{bmatrix} \overline{\mathbf{4}} \cdot \mathbf{E} - 5 & 0 & 0 \\ 0 & 500 & 0 \\ 0 & 0 & 100 \end{bmatrix}$$

This set of plots was generated by setting the mass in the truth model to 2 slugs. The fourth order filter was used with the mass of the missile in its model set at 4. Both, filter and truth model used an S (for M/S) of .137 ft<sup>2</sup>. Only the dynamic states of the missile model were estimated.

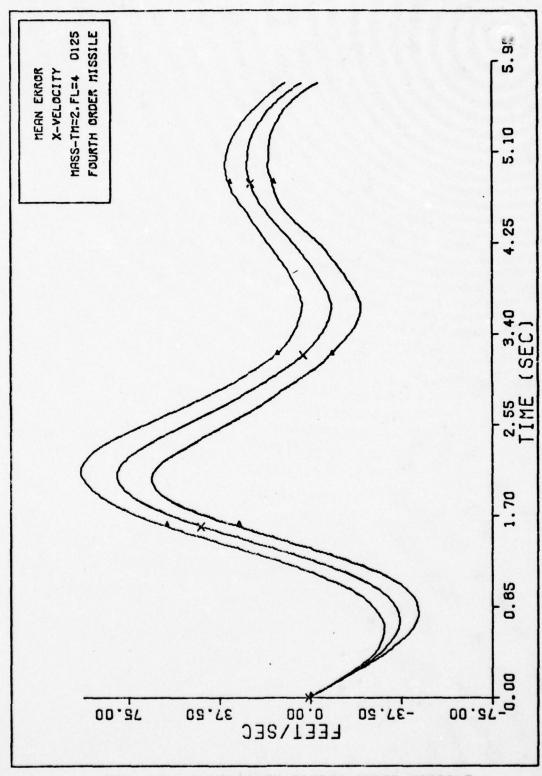


Fig. 122. X-VELOCITY FOURTH ORDER MISSILE

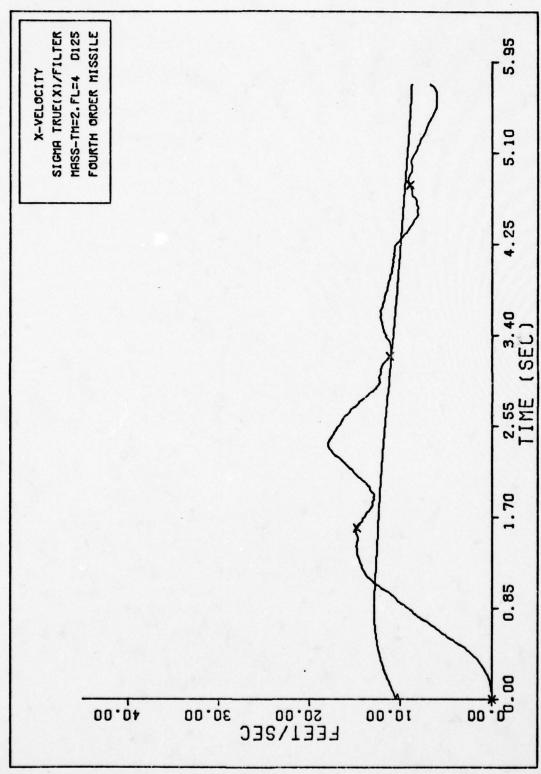


Fig. 123. X-VELOCITY SIGMAS FOURTH ORDER

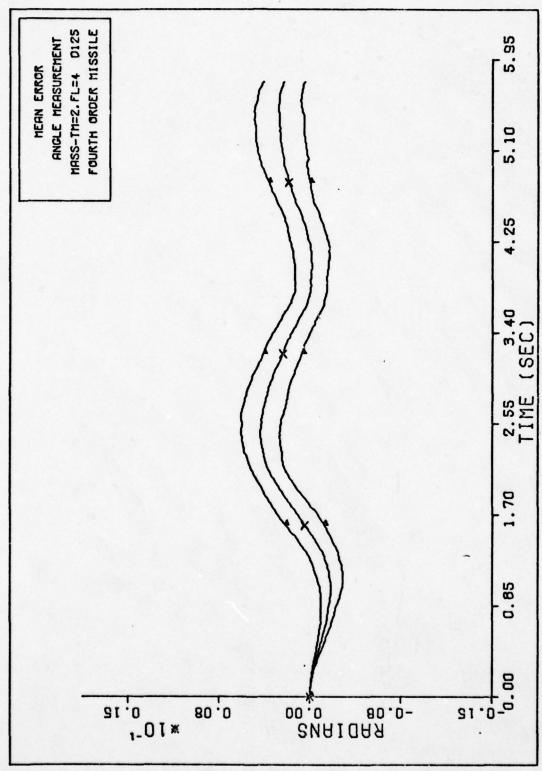


Fig. 124. ANGLE MEASUREMENT FOURTH ORDER MISSILE

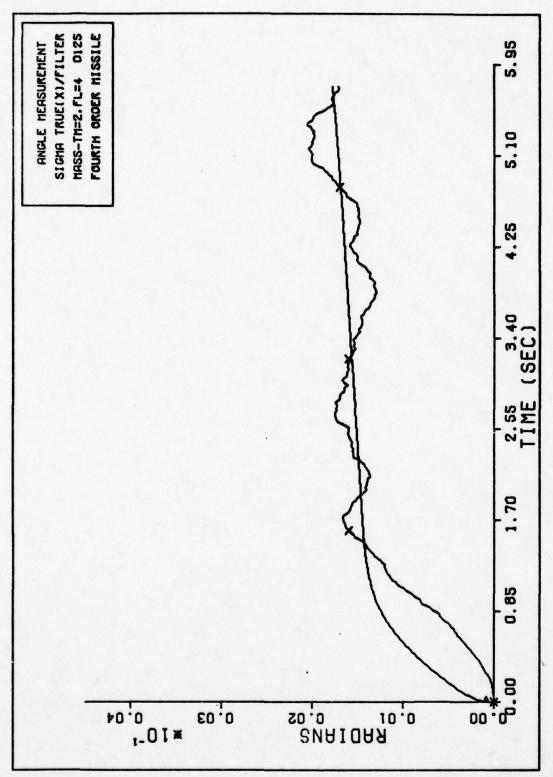


Fig. 125. ANGLE MEASUREMENT SIGMAS FOURTH ORDER

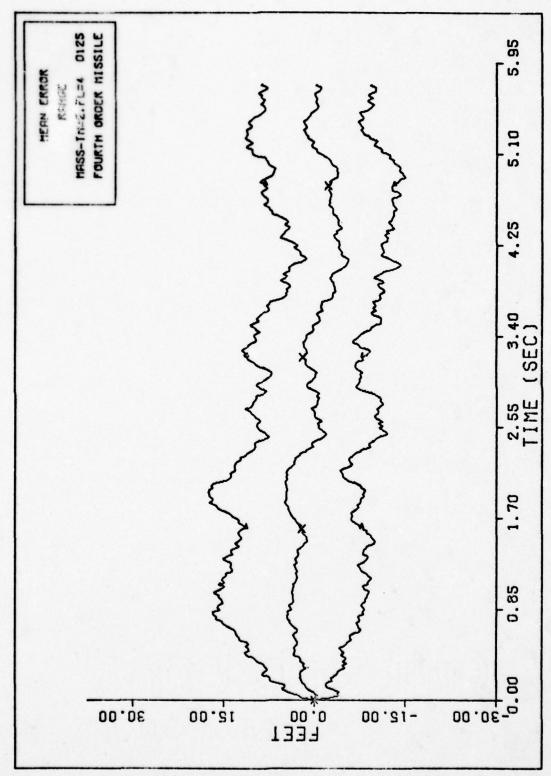


Fig. 126. RANGE FOURTH ORDER MISSILE

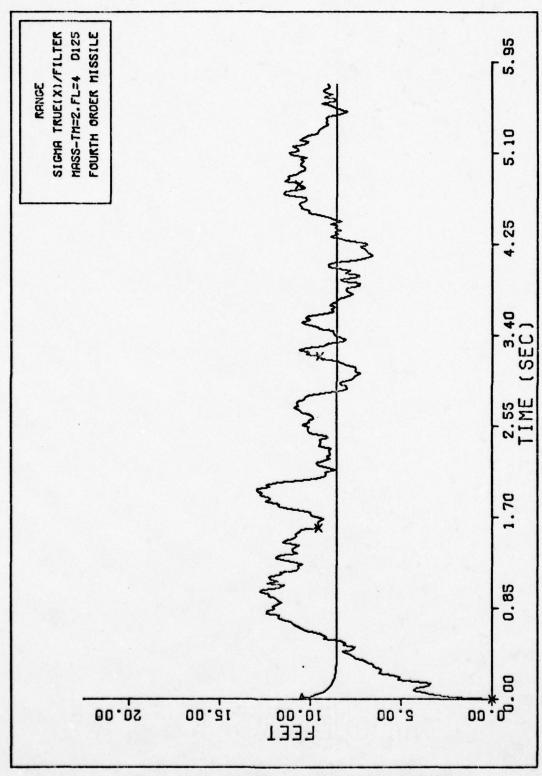


Fig. 127. RANGE SIGMAS FOURTH ORDER

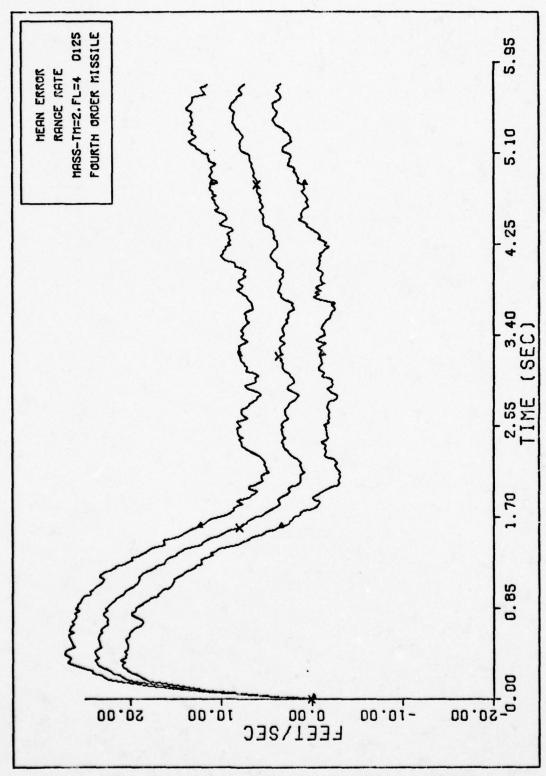


Fig. 128. RANGE RATE FOURTH ORDER MISSILE

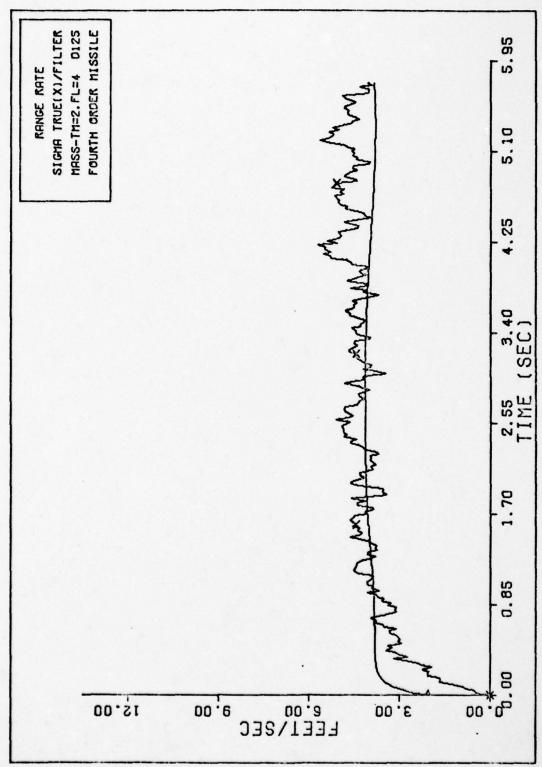


Fig. 129. RANGE RATE SIGMAS FOURTH ORDER

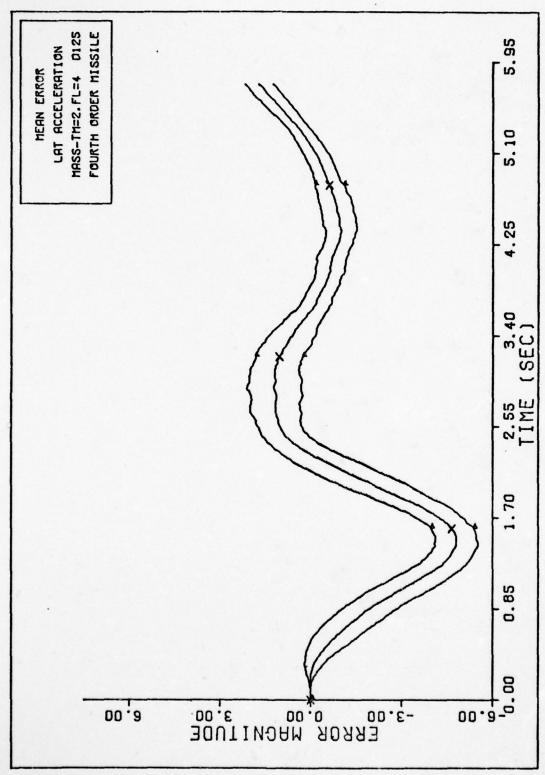


Fig. 130. LAT ACCELERATION FOURTH ORDER MISSILE

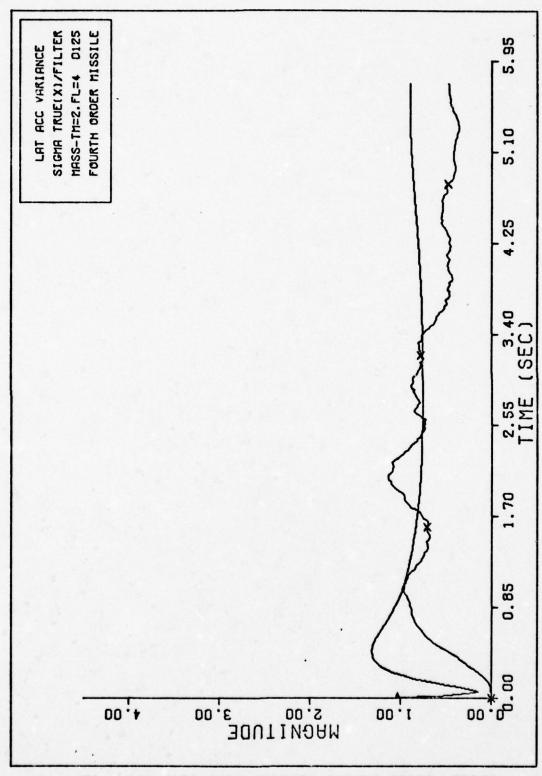


Fig. 131. LAT ACCELERATION SIGMAS FOURTH ORDER

## Sensitivity Analysis (M = 8.)

The initial state estimates and the tuning parameters for this case are

$$V_{mx}^{I}(0) = 1225.7 \text{ fps}$$
 $\dot{\theta}_{I}(0) = 4.363345 \text{ radians}$ 
 $R(0) = 10000. \text{ feet}$ 
 $\dot{R}(0) = -2122. \text{ fps}$ 
 $a_{L}(0) = 0.$ 
 $n(0) = 4.5$ 
 $\tau_{f}(0) = N/A$ 
 $M/S(0) = 29.197 \text{ slugs/ft}^{2}$ 

$$\mathbf{R} = \begin{bmatrix} 4.E-5 & 0. & 0. \\ 0. & 500. & 0. \\ 0. & 0. & 100. \end{bmatrix}$$

This set of plots was generated by setting the mass in the truth model to 8 slugs. The fourth order filter was used with the mass of the missile in its model set at 4.

Both, filter and truth model used an S (for M/S) of .137 ft<sup>2</sup>. Only the dynamic states of the missile model were estimated.

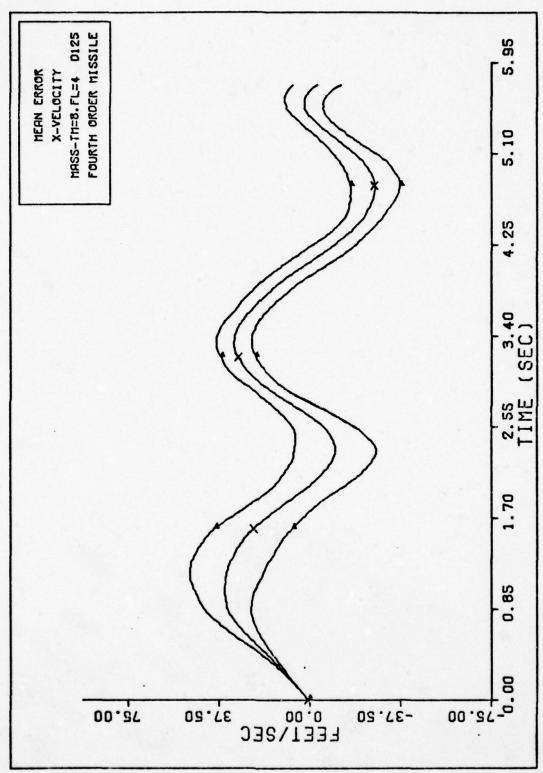


Fig. 132. X-VELOCITY FOURTH ORDER MISSILE

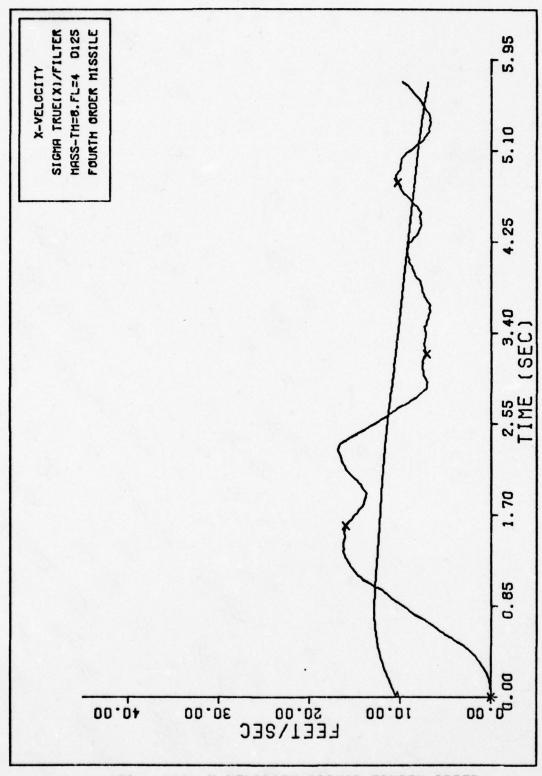


Fig. 133. X-VELOCITY SIGMAS FOURTH ORDER

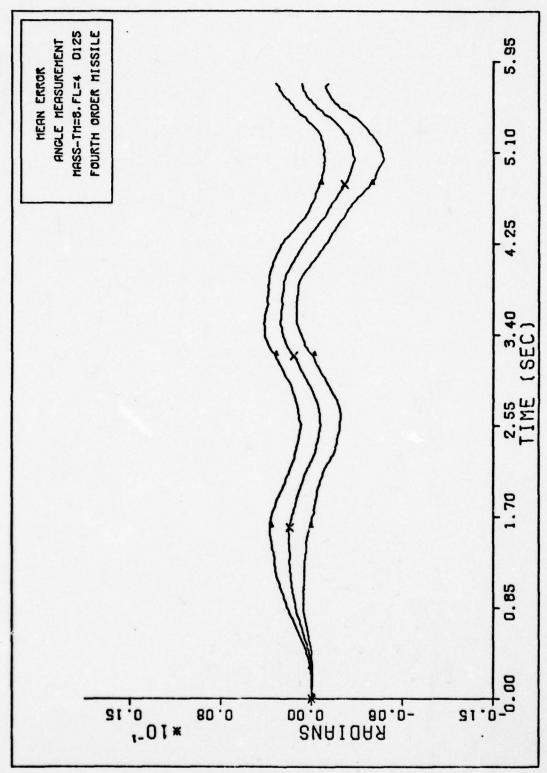


Fig. 134. ANGLE MEASUREMENT FOURTH ORDER MISSILE

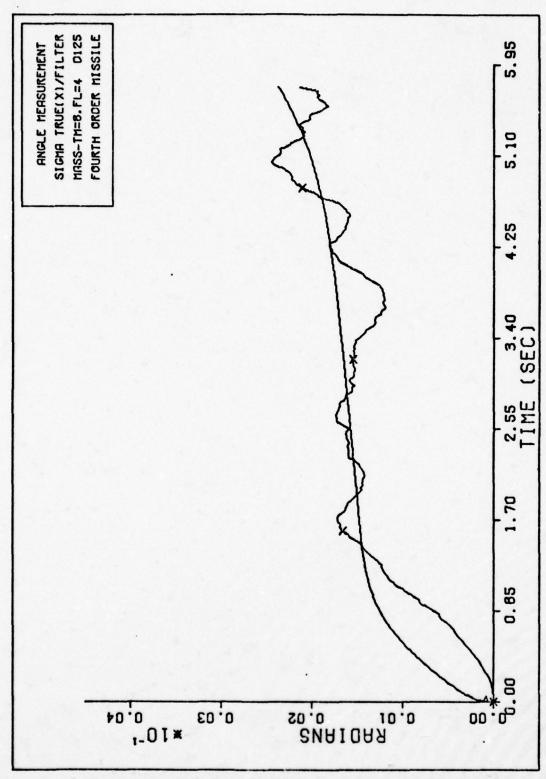


Fig. 135. ANGLE MEASUREMENT SIGMAS FOURTH ORDER

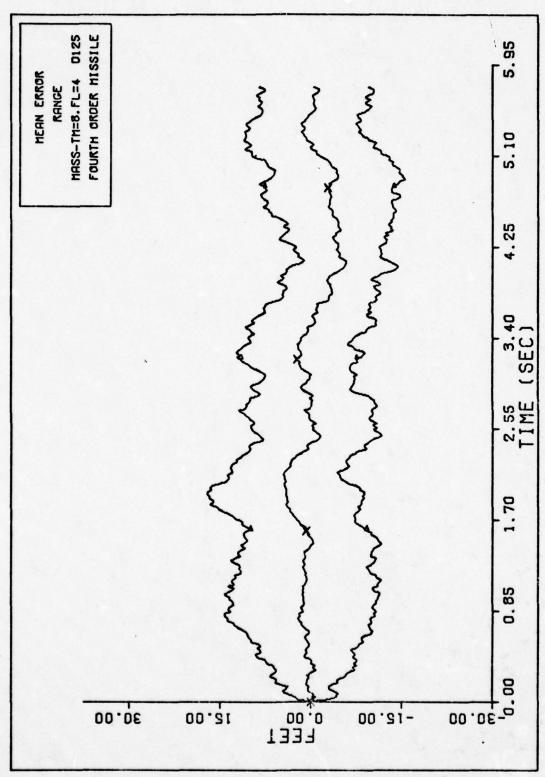


Fig. 136. RANGE FOURTH ORDER MISSILE

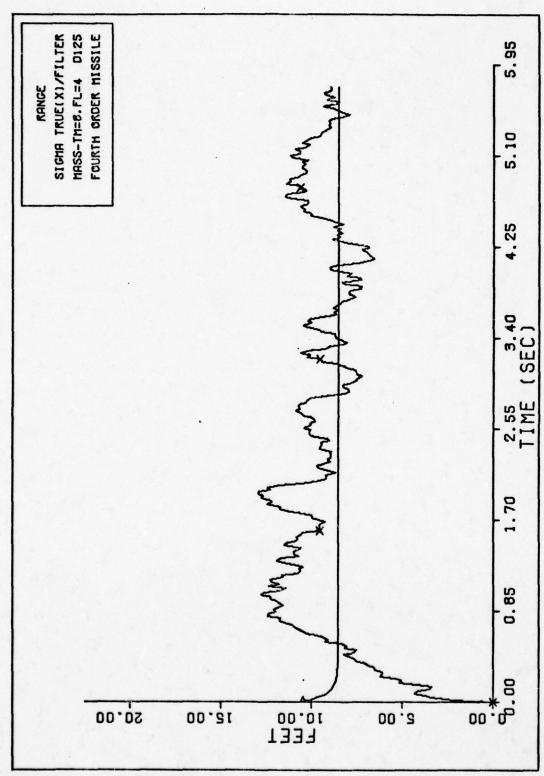


Fig. 137. RANGE SIGMAS FOURTH ORDER

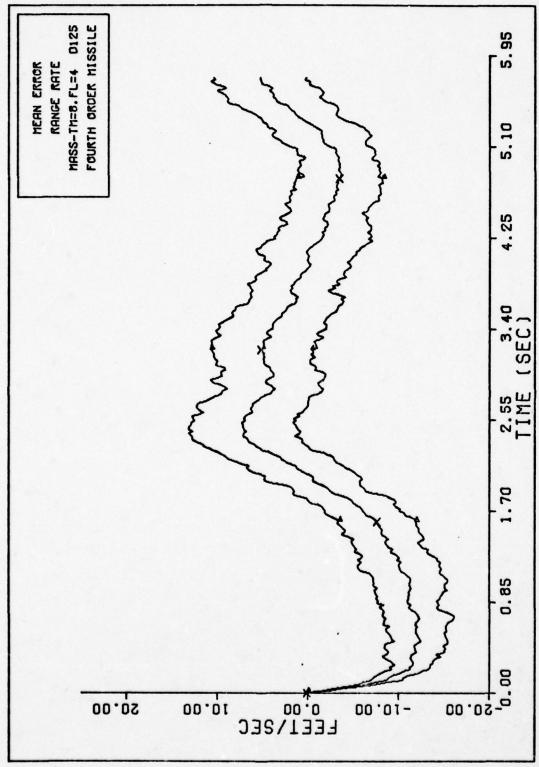


Fig. 138. RANGE RATE FOURTH ORDER MISSILE

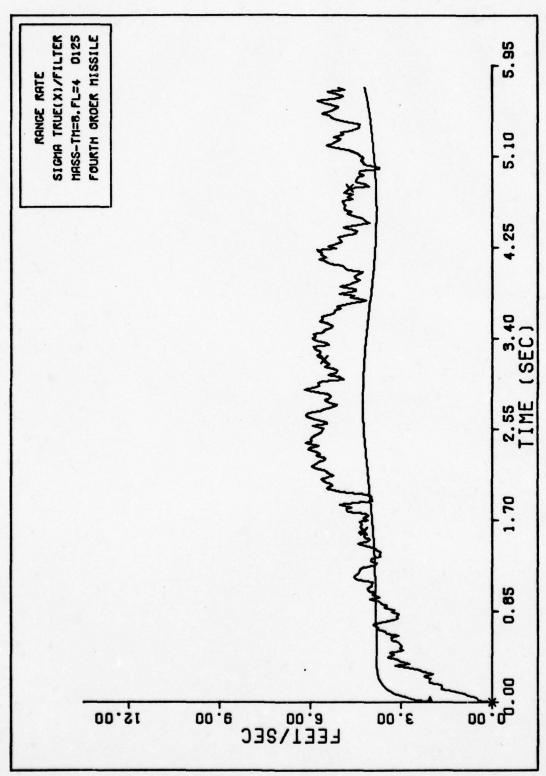


Fig. 139. RANGE RATE SIGMAS FOURTH ORDER

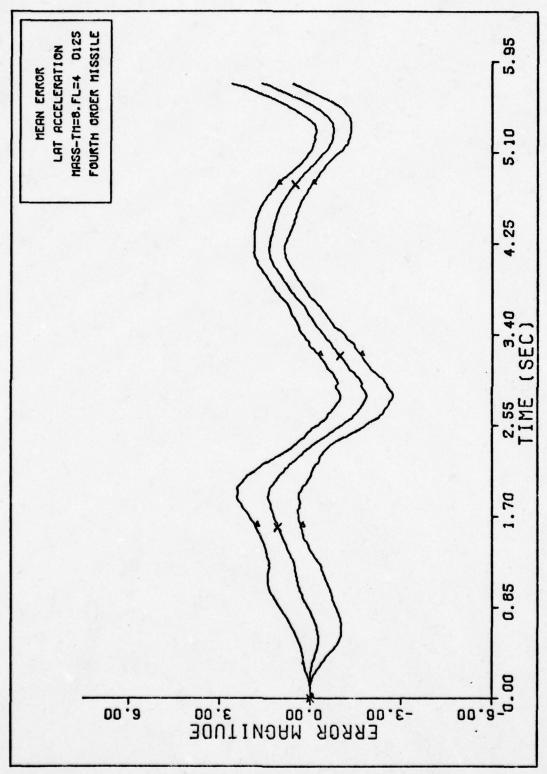


Fig. 140. LAT ACCELERATION FOURTH ORDER MISSILE

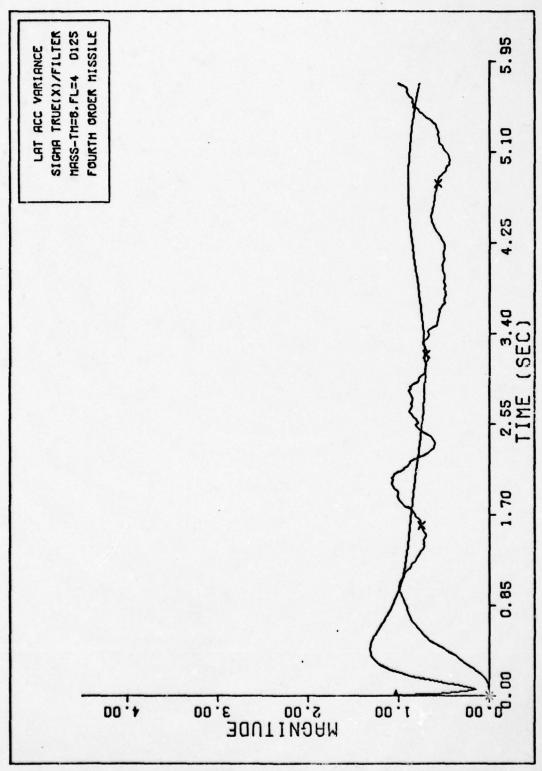


Fig. 141. LAT ACCELERATION SIGMAS FOURTH ORDER

## First Order Missile Filter (Tf set equal to .85)

The initial state estimates and the tuning parameters for this case are

$$v_{mx}^{I}(0) = 1225.7 \text{ fps}$$

$$e(0) = 4.363345 \text{ radians}$$

$$\dot{R}(0) = -2122. \text{ fps}$$

$$a_{T_i}(0) = 0.$$

$$n(0) = 4.5$$

$$\tau_{\mathbf{f}}(0) = .85 \text{ seconds}$$

$$M/S(0) = 29.197 \text{ slugs/ft}^2$$

$$\underline{\mathbf{R}} = \begin{bmatrix} \overline{\mathbf{3}}.\mathbf{E} - 5 & 0. & 0. \\ 0. & 500. & 0. \\ 0. & 0. & 100. \end{bmatrix}$$

These plots were generated by the first order filter with a time constant of .85 seconds. The value of  $\tau_{\mathbf{f}}$  was found from an iterative process by comparing filter performance for various values of  $\tau_{\mathbf{f}}$ . .85 seconds was found to produce the least error in the dynamic states of the missile model. Only the dynamic states were estimated in this case.

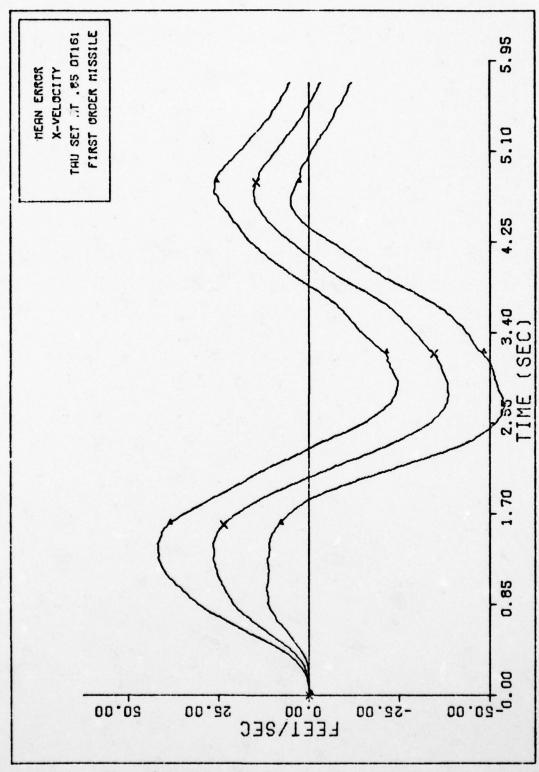


Fig. 142. X-VELOCITY FIRST ORDER MISSILE

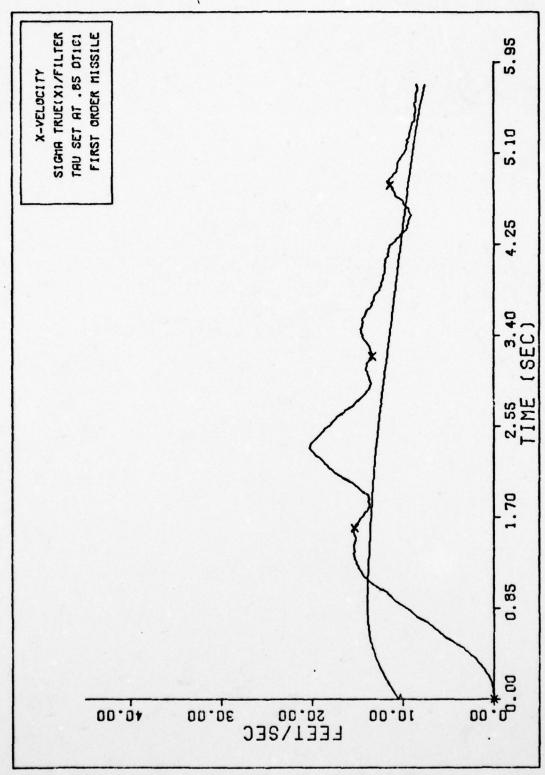


Fig. 143. X-VELOCITY SIGMAS FIRST ORDER

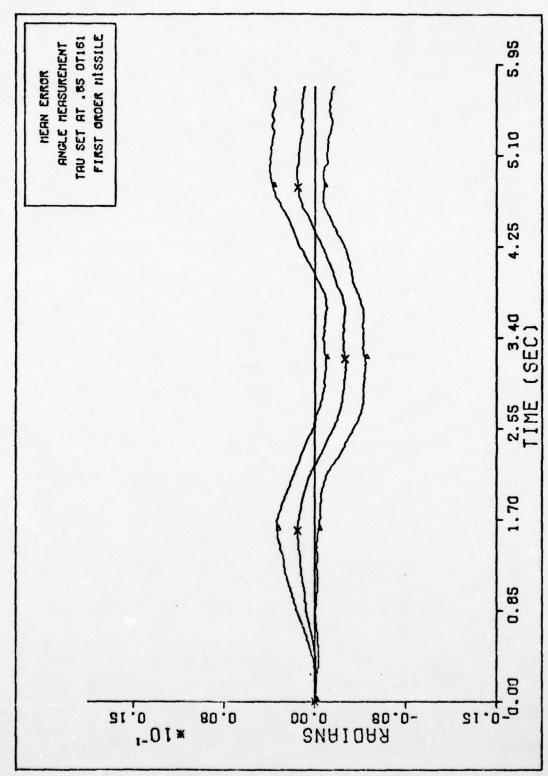


Fig. 144. ANGLE MEASUREMENT FIRST ORDER MISSILE

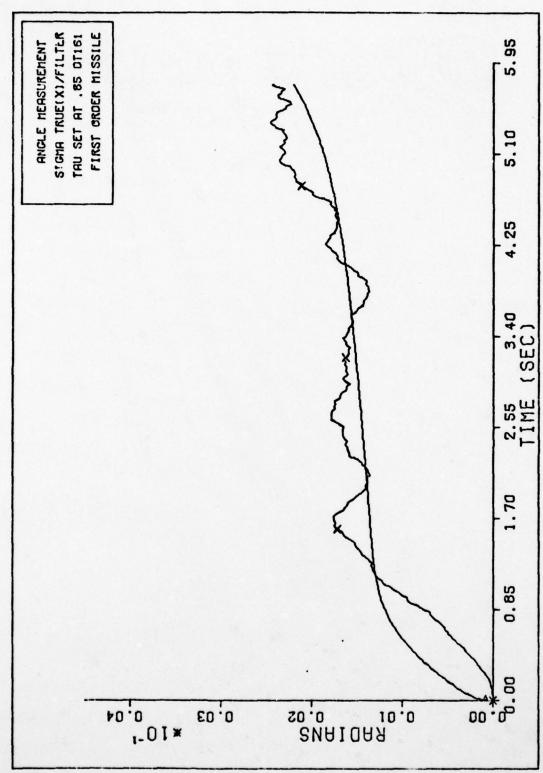


Fig. 145. ANGLE MEASUREMENT SIGMAS FIRST ORDER

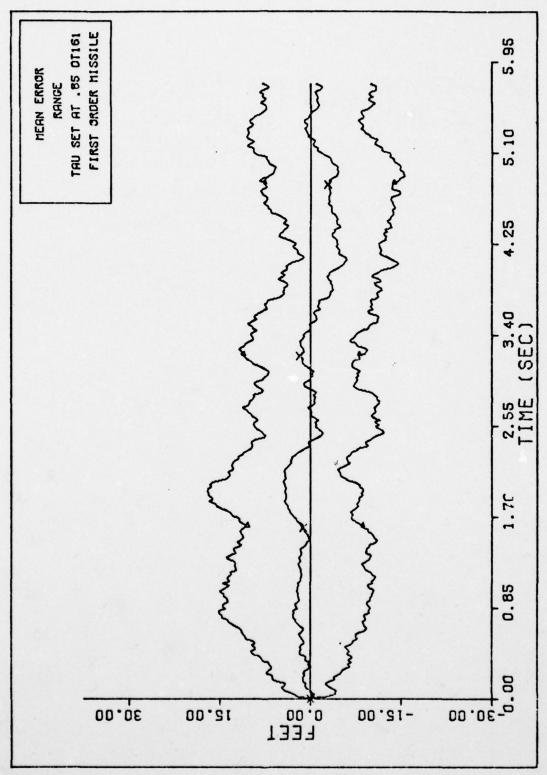


Fig. 146. RANGE FIRST ORDER MISSILE

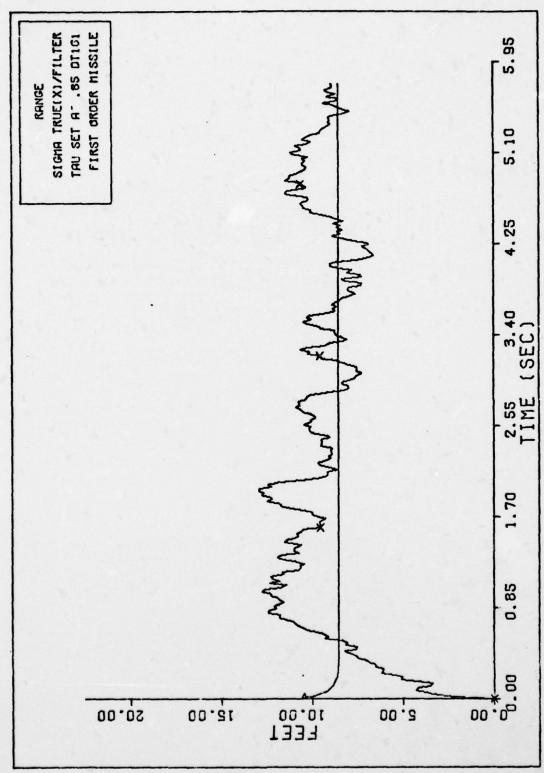


Fig. 147. RANGE SIGMAS FIRST ORDER

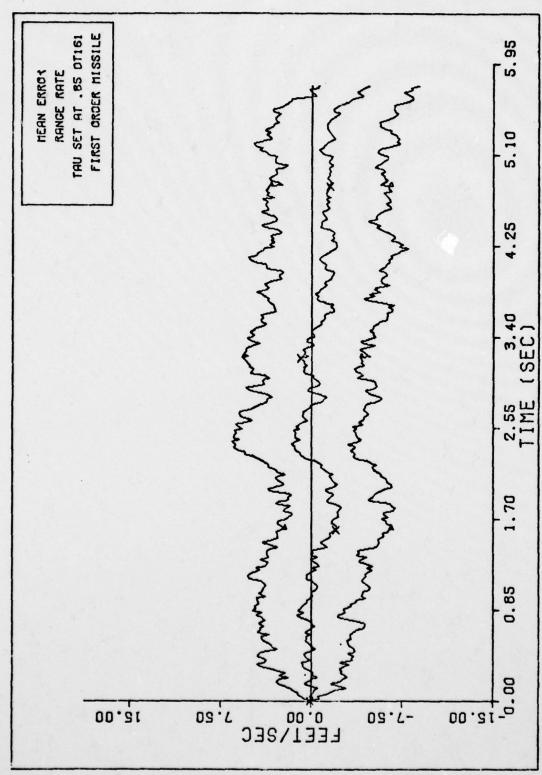


Fig. 148. RANGE RATE FIRST ORDER MISSILE

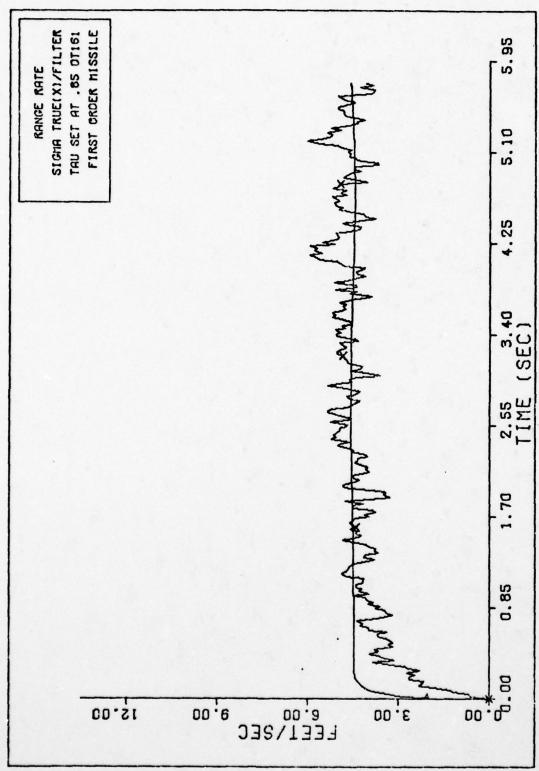


Fig. 149. RANCE RATE SIGMAS FIRST ORDER

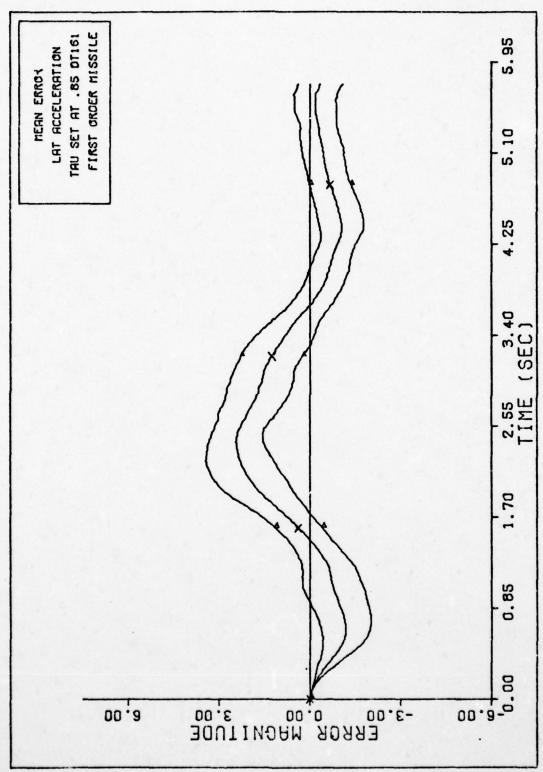


Fig. 150. LAT ACCELERATION FIRST ORDER MISSILE

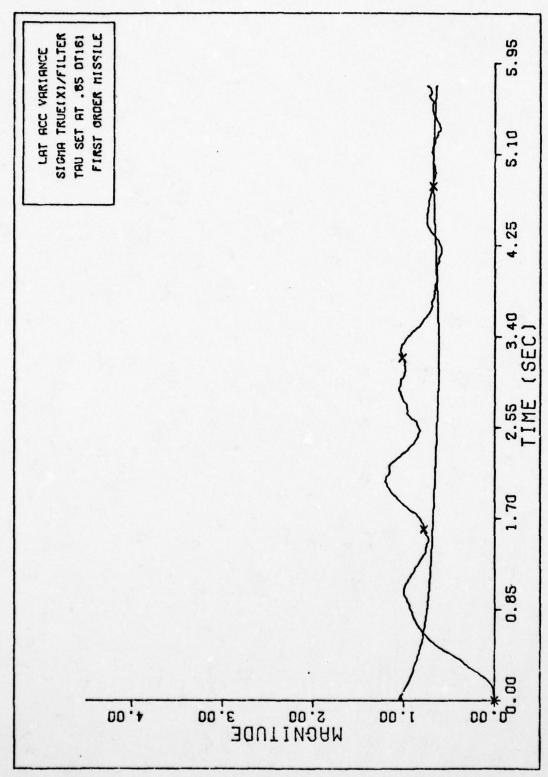


Fig. 151. LAT ACCELERATION SIGMAS FIRST ORDER

## n Estimation - n Initialized at 3.

The initial state estimates and the tuning parameters for this case are

$$V_{mx}^{I}(0) = 1225.7 \text{ fps}$$
 $\dot{Q}(0) = 4.363345 \text{ radians}$ 
 $R(0) = 10000. \text{ feet}$ 
 $\dot{R}(0) = -2122. \text{ fps}$ 
 $a_{L}(0) = 0.$ 
 $n(0) = 3.$ 
 $\tau_{f}(0) = .85 \text{ seconds}$ 
 $M/S(0) = 29.197 \text{ slugs/ft}^{2}$ 

$$\underline{\mathbf{R}} = \begin{bmatrix} \overline{\mathbf{3.E-5}} & 0. & 0. \\ 0. & 500. & 0. \\ 0. & 0. & 100. \end{bmatrix}$$

This set of plots was generated by the first order filter when estimating n, which was initialized at 3 in the filter. The true value of n in the truth model was 4.5. The other parameters in the filter model were not estimated.

AIR FORCE INST OF TECH WRIGHT-PATTERSON AFB OHIO SCH-ETC F/G 19/5
AN EXTENDED KALMAN FILTER FIRE CONTROL SYSTEM AGAINST AIR-TO-AI--ETC(U)
DEC 77 S J CUSUMANO, M DE PONTE
AFIT/GE/EE/77-13-VOL-2
NL AD-A055 637 UNCLASSIFIED 3 OF 4 ADA 055637 ### ### Ú

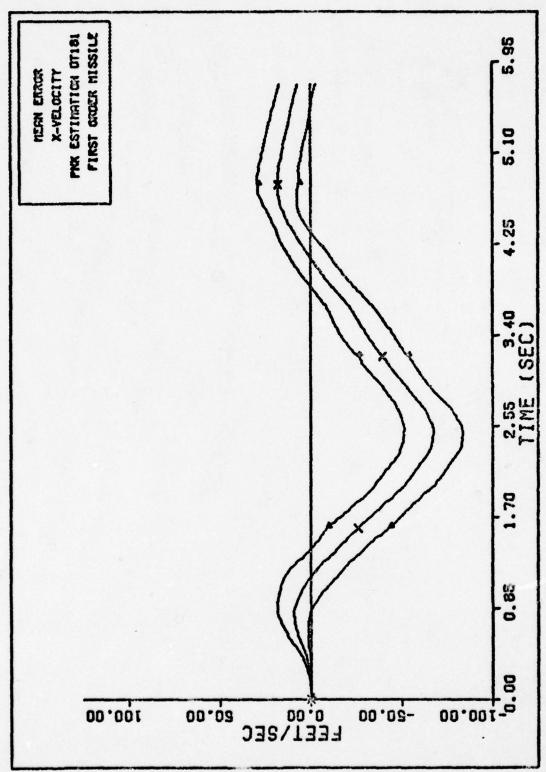


Fig. 152. X-VELOCITY FIRST ORDER MISSILE

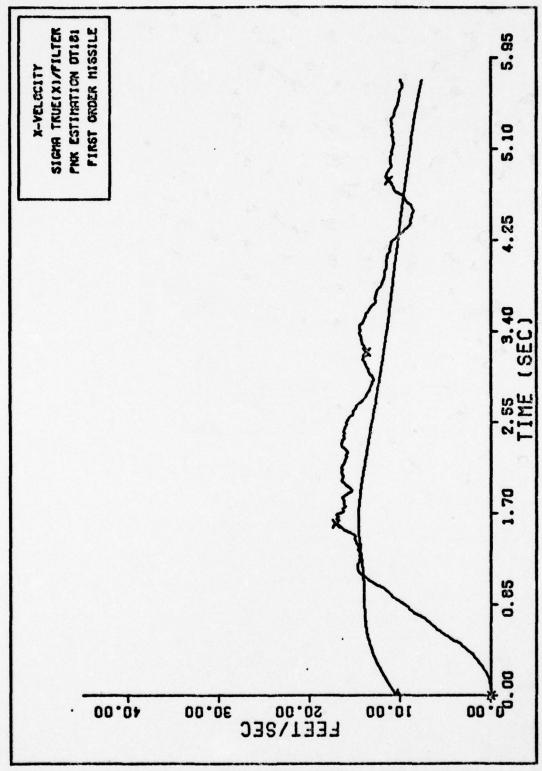


Fig. 153. X-VELOCITY SIGMAS FIRST ORDER

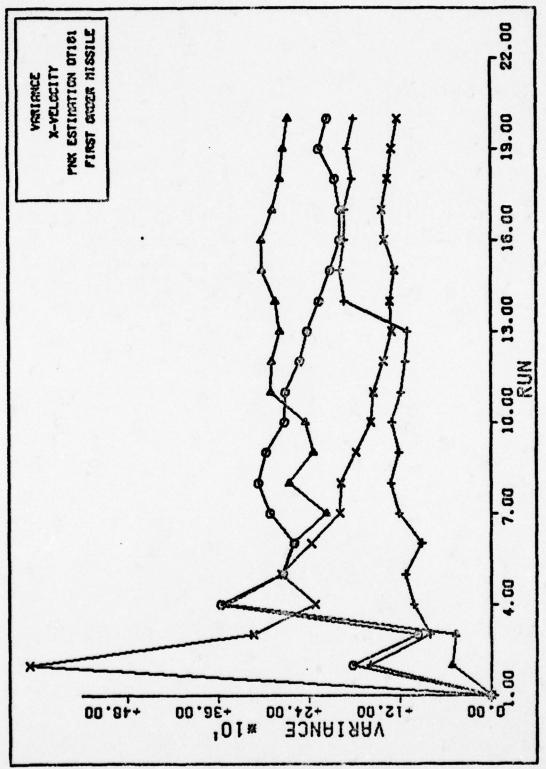


Fig. 154. VARIANCE CONVERGENCE

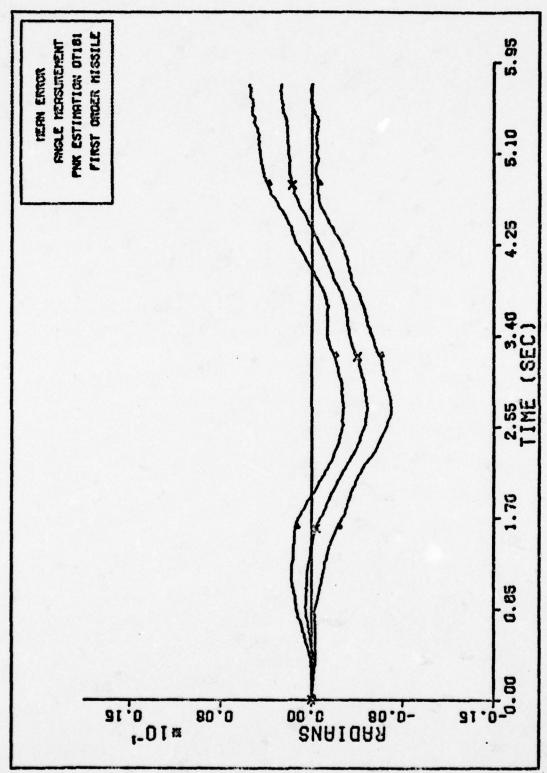


Fig. 155. ANGLE MERSUREMENT FIRST ORDER MISSILE

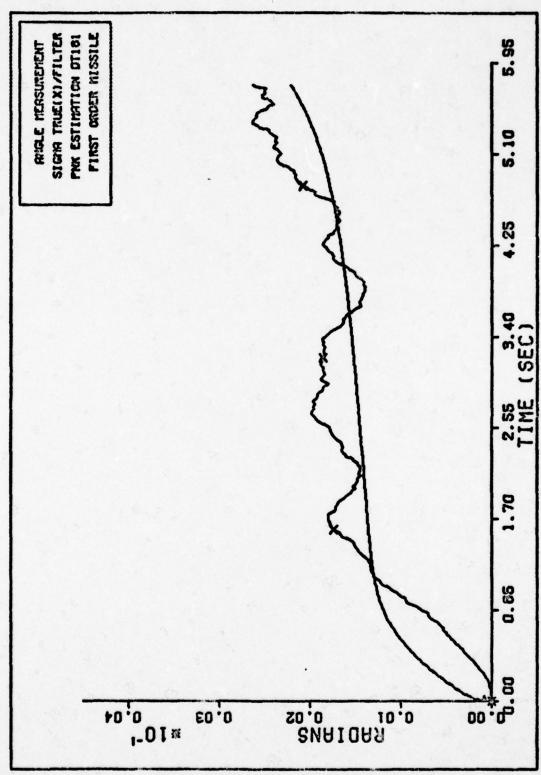


Fig. 156. ANGLE MERSUREMENT SIGMAS FIRST ORDER

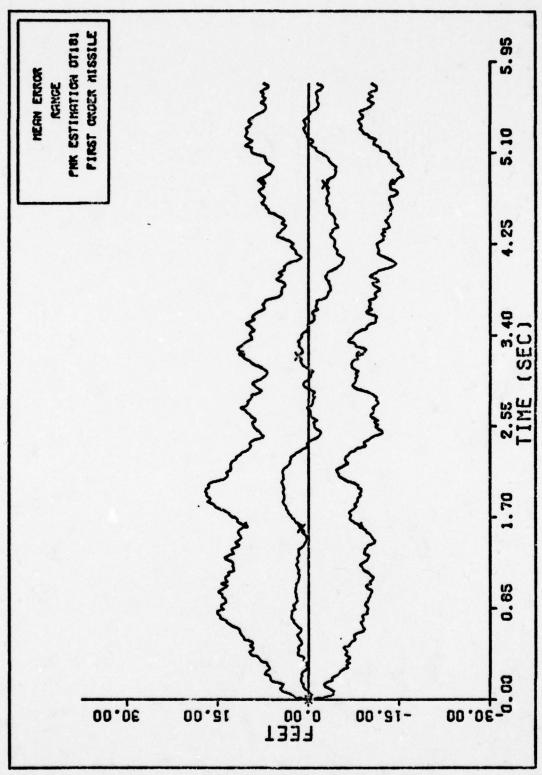


Fig. 157. RANGE FIRST ORDER MISSILE

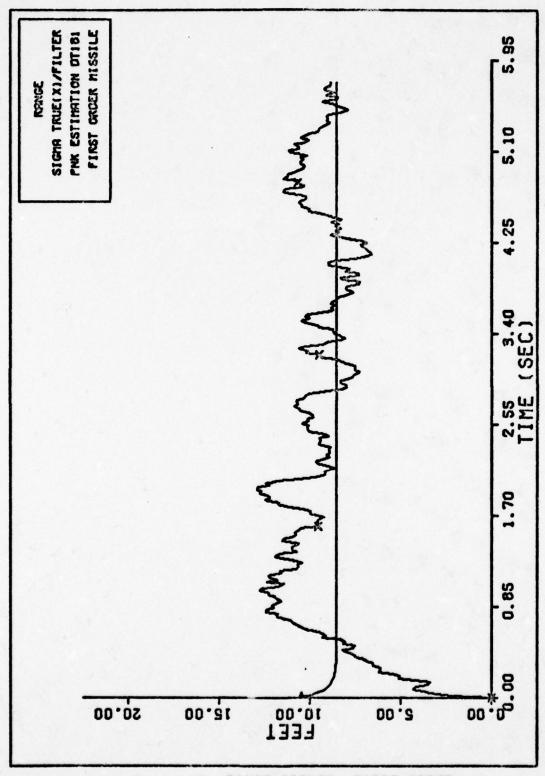


Fig. 158. RANGE SIGMAS FIRST ORDER

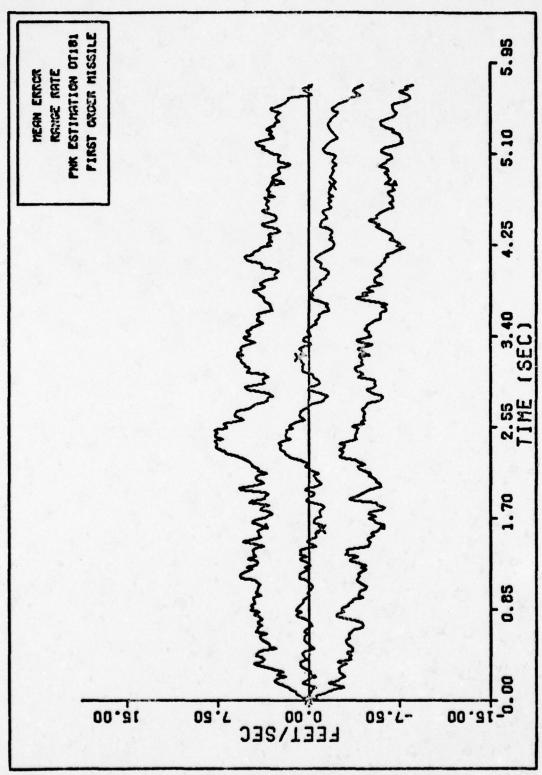


Fig. 159. RANGE RATE FIRST ORDER MISSILE

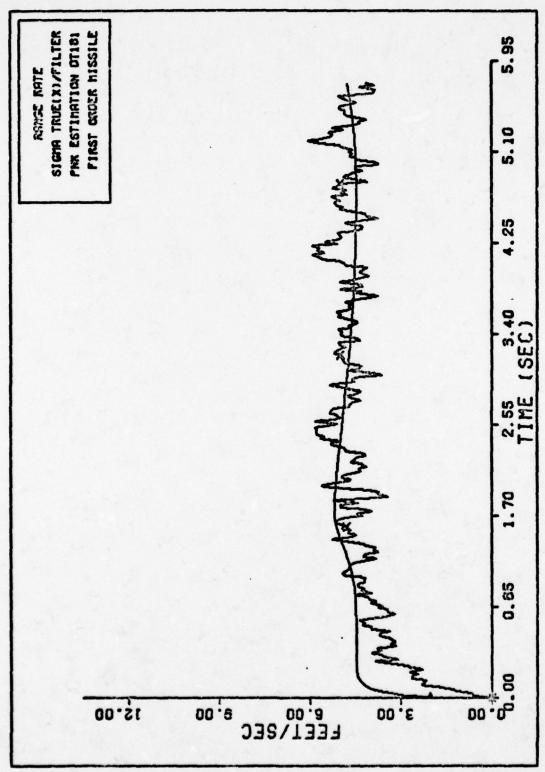


Fig. 160. RANGE RATE SIGMAS FIRST ORDER

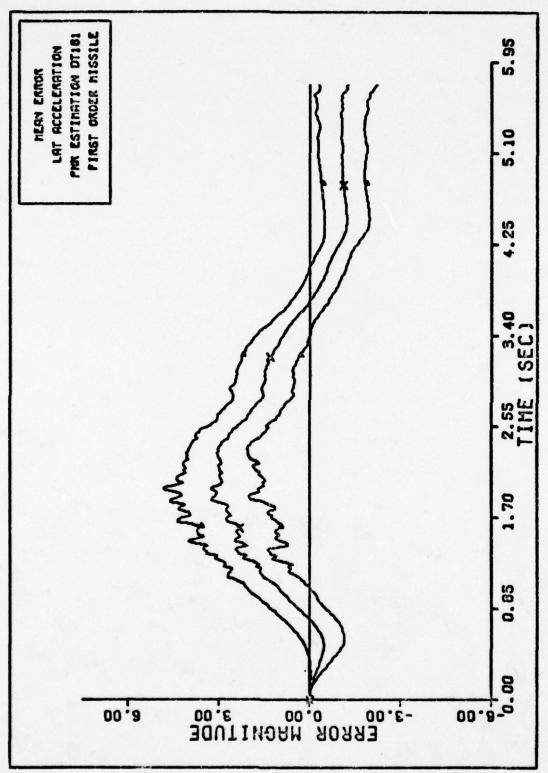


Fig. 161. LAT ACCELERATION FIRST ORDER MISSILE

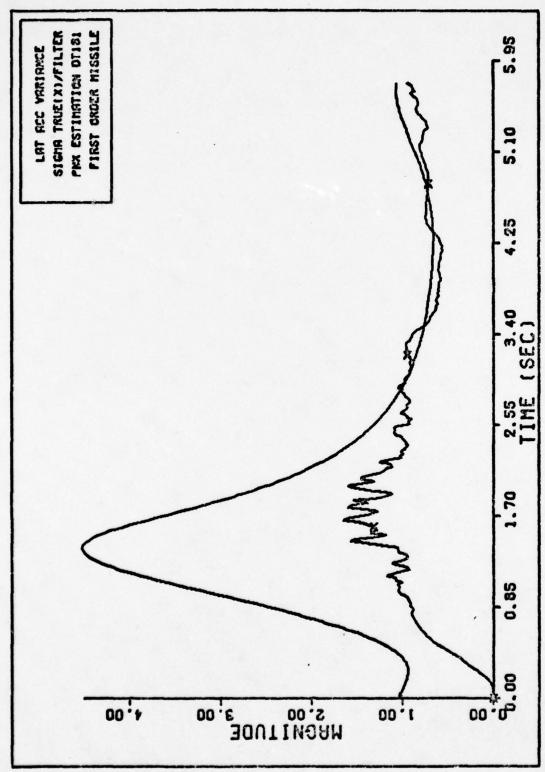


Fig. 162. LAT ACCELERATION SIGMAS FIRST ORDER

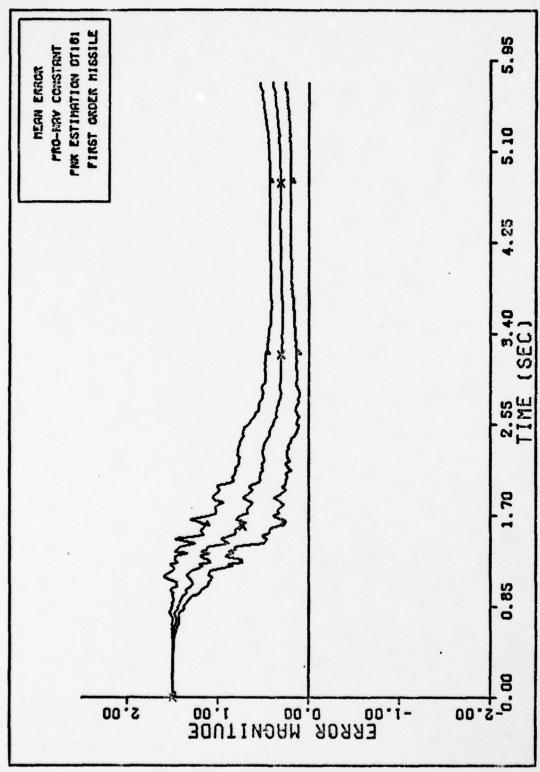


Fig. 163. PRO-NAV CONSTANT FIRST ORDER MISSILE

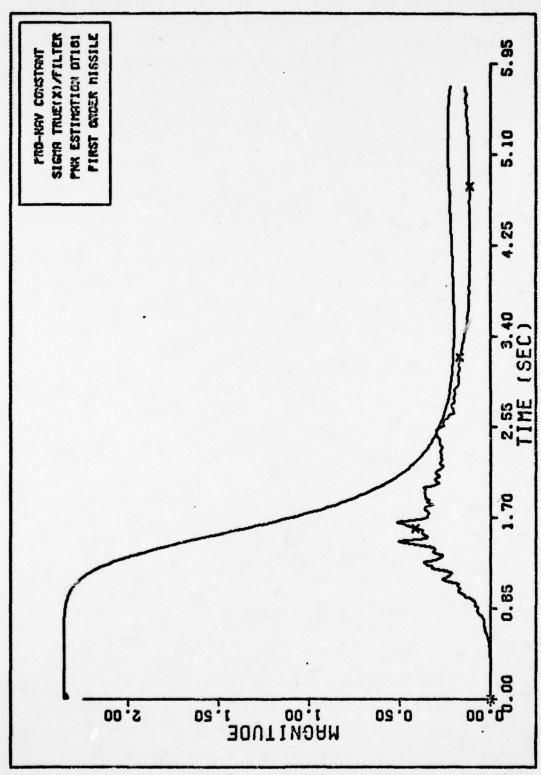


Fig. 164. PRO-NAY CONSTANT SIGMAS FIRST ORDER

## n Estimation - n Initialized at 6.

The initial state estimates and the tuning parameters for this case are

$$v_{mx}^{I}(0) = 1225.7 \text{ fps}$$
 $\dot{\theta}(0) = 4.363345 \text{ radians}$ 
 $R(0) = 10000. \text{ feet}$ 
 $\dot{R}(0) = -2122. \text{ fps}$ 
 $a_{L}(0) = 0.$ 
 $n(0) = 6.$ 
 $\tau_{f}(0) = .85 \text{ seconds}$ 
 $M/S(0) = 29.197 \text{ slugs/ft}^{2}$ 

$$\mathbf{R} = \begin{bmatrix} 3.E-5 & 0. & 0. \\ 0. & 500. & 0. \\ 0. & 0. & 100. \end{bmatrix}$$

This set of plots was generated by the first order filter when estimating n, which was initialized at 6 in the filter. The true value of n in the truth model was 4.5. The other parameters in the filter model were not estimated.

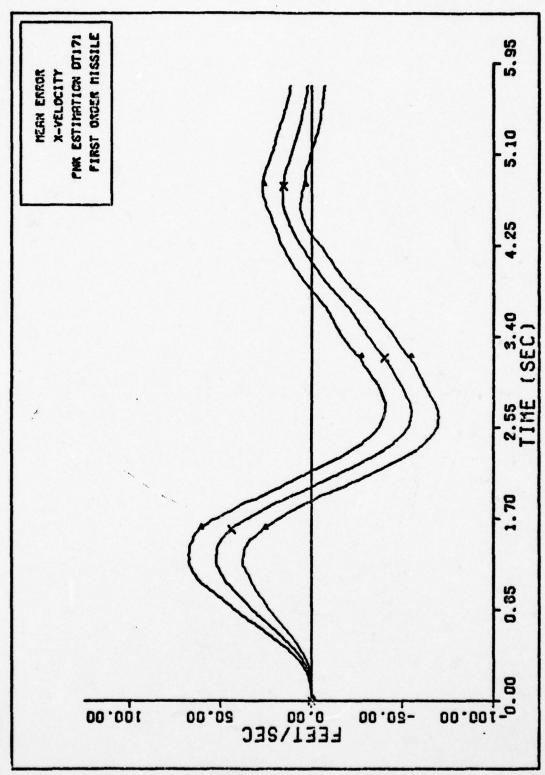


Fig. 165. X-VELOCITY FIRST ORDER MISSILE

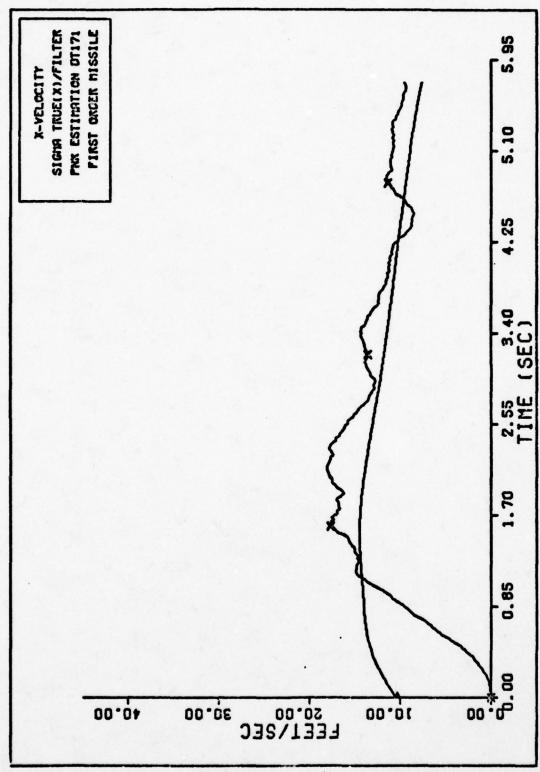


Fig. 166. X-VELOCITY SIGMAS FIRST ORDER

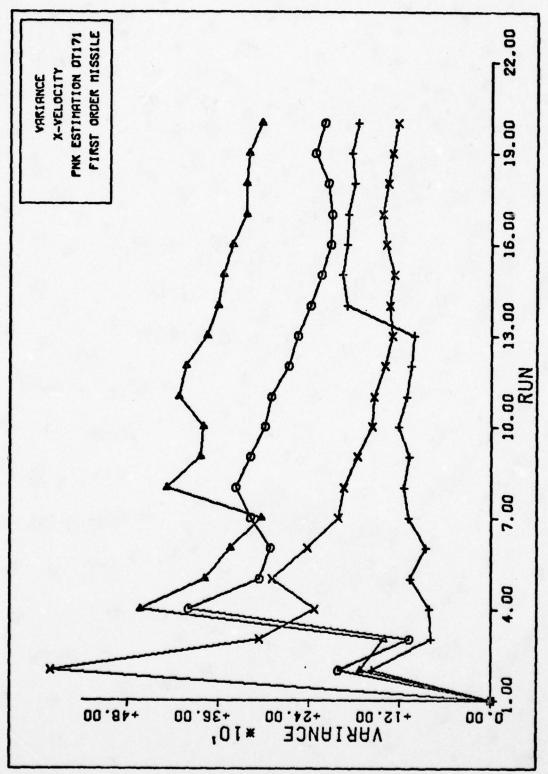


Fig. 167. VARIANCE CONVERGENCE

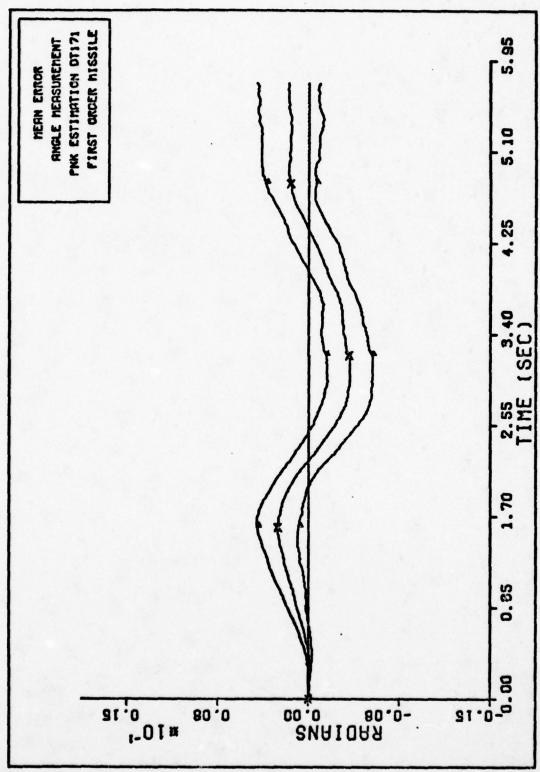


Fig. 168. ANGLE MEASUREMENT FIRST ORDER MISSILE

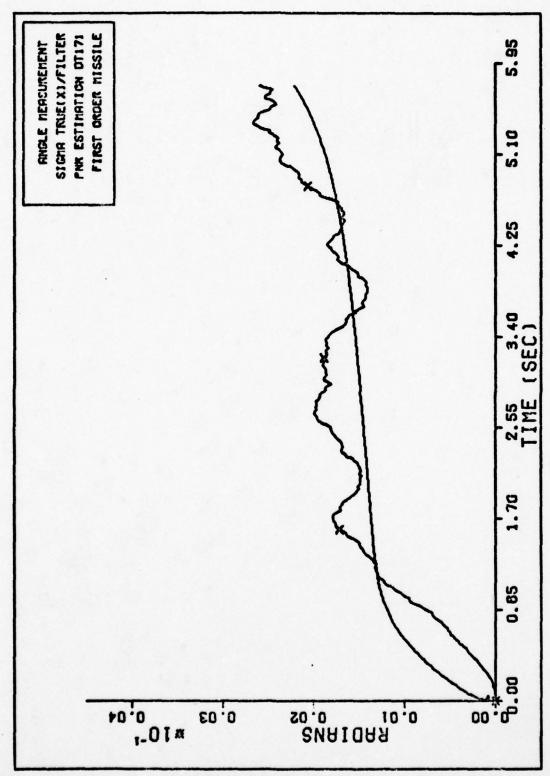


Fig. 169. ANGLE MEASUREMENT SIGNAS FIRST ORDER

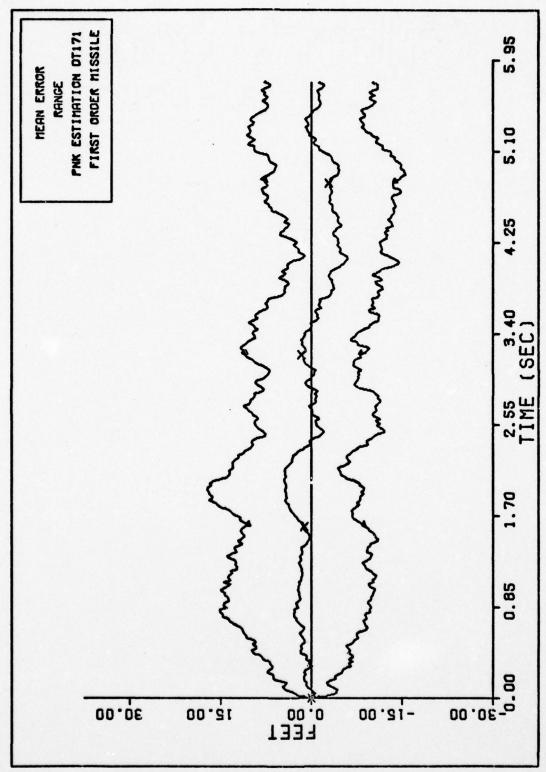


Fig. 170. RANGE FIRST ORDER MISSILE

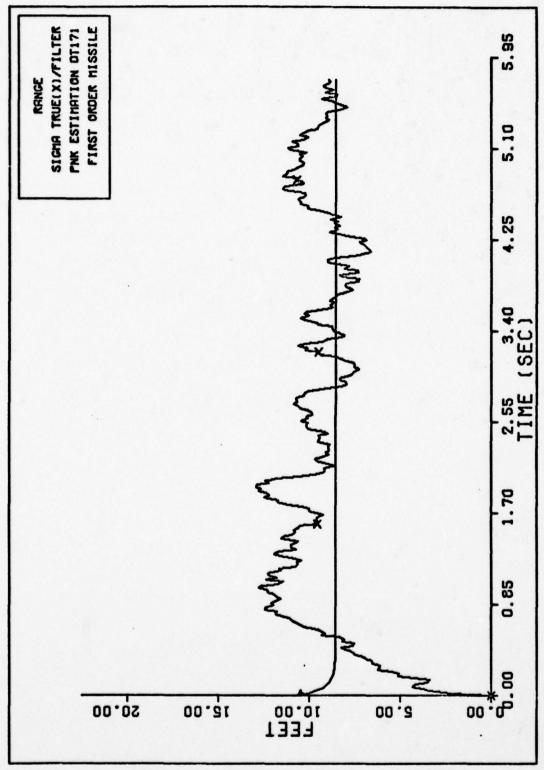


Fig. 171. RANGE SIGMAS FIRST ORDER

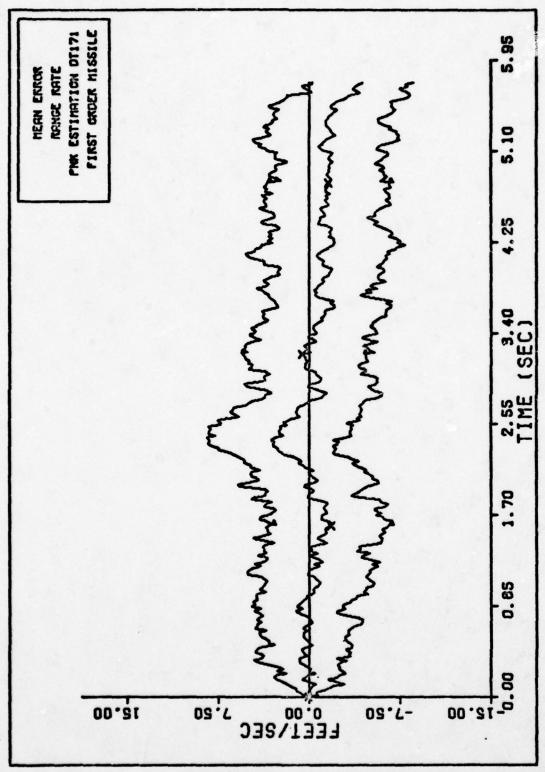


Fig. 172. RANGE RATE FIRST ORDER MISSILE

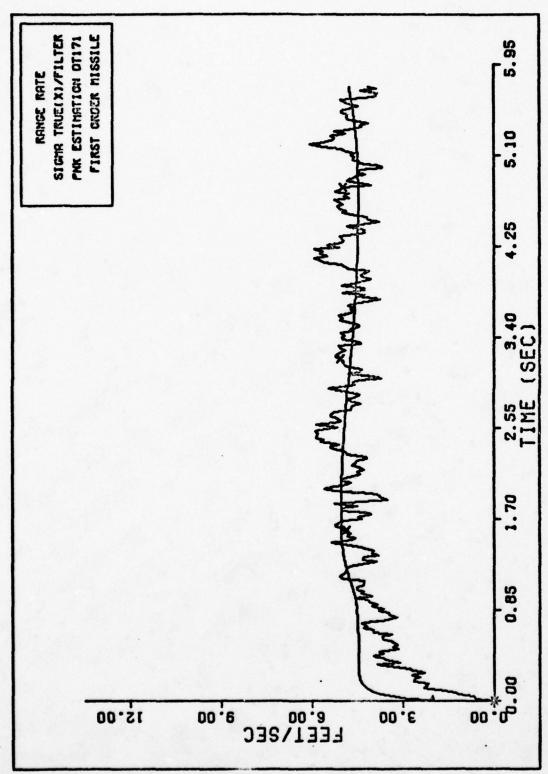


Fig. 173. RANGE RATE SIGNAS FIRST ORDER

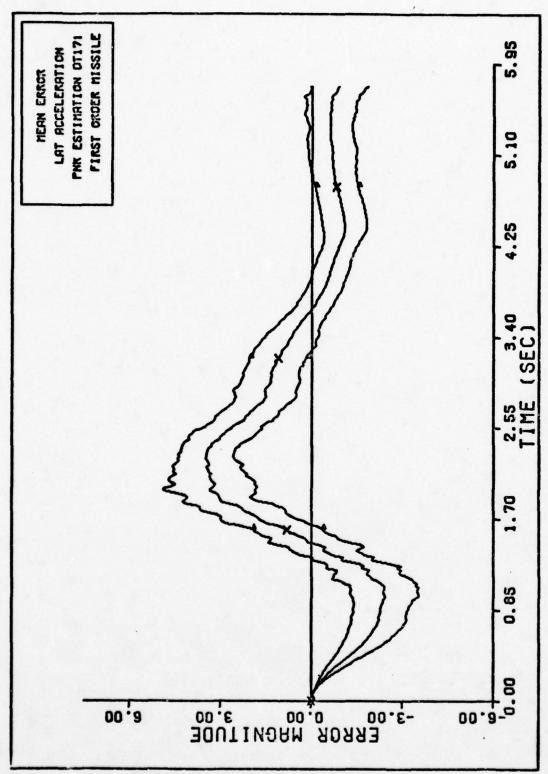


Fig. 174. LAT ACCELERATION FIRST ORDER MISSILE

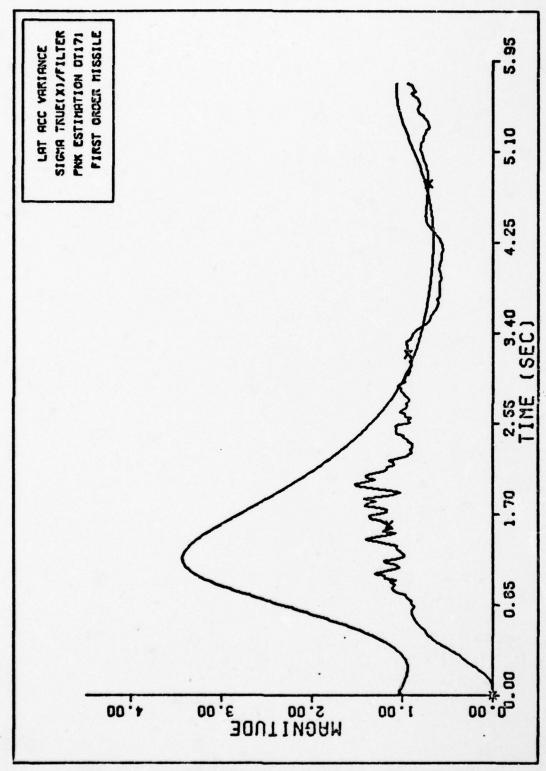


Fig. 175. LAT ACCELERATION SIGMAS FIRST ORDER

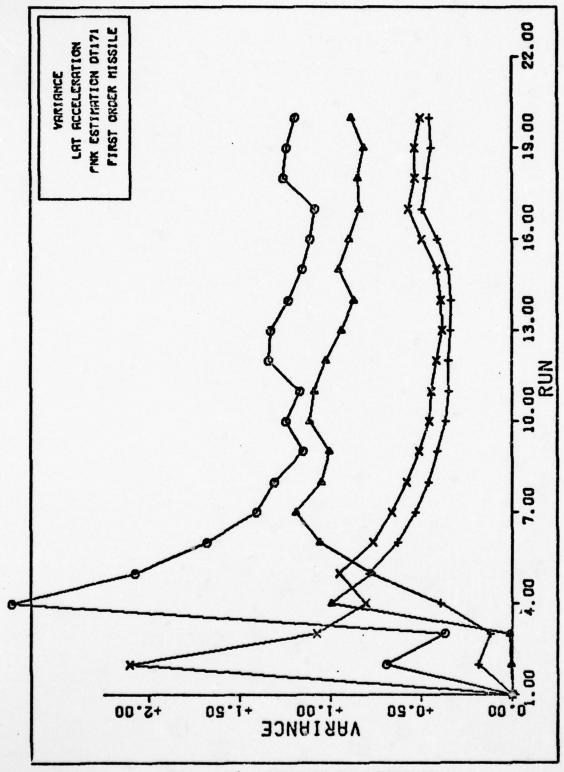


Fig. 176. VARIANCE CONVERGENCE

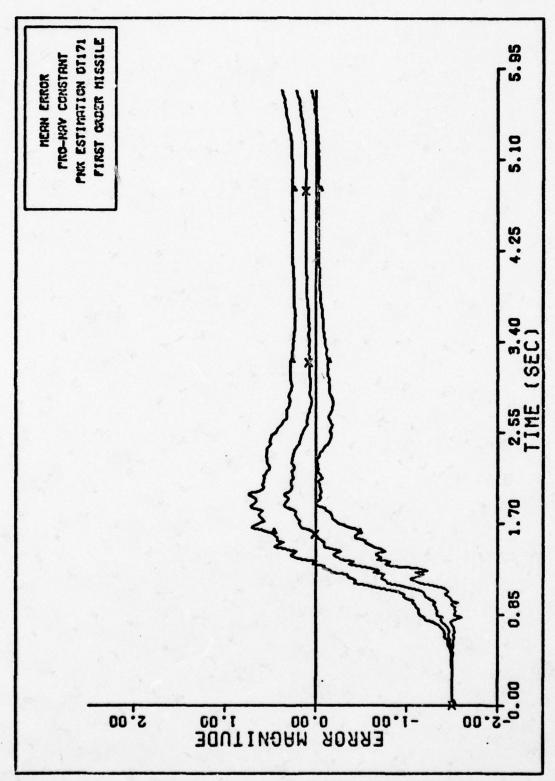


Fig. 177. PRO-NAY CONSTANT FIRST ORDER MISSILE

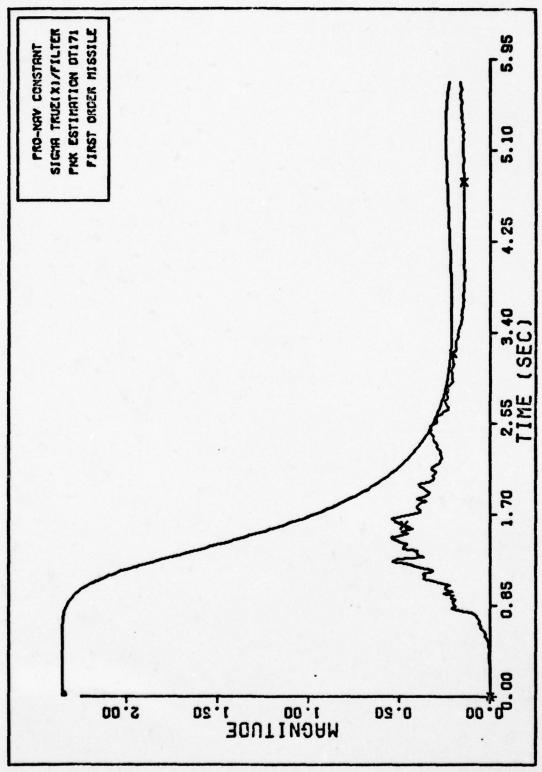


Fig. 178. PRO-NAV CONSTRNT SIGNAS FIRST ORDER

## If Estimation - If initialized at 1.5

The initial state estimates and the tuning parameters for this case are

$$v_{mx}^{I}(0) = 1225.7 \text{ fps}$$
 $v_{mx}^{I}(0) = 4.363345 \text{ radians}$ 
 $v_{mx}^{I}(0) = 4.363345 \text{ radians}$ 
 $v_{mx}^{I}(0) = 10000. \text{ feet}$ 
 $v_{mx}^{I}(0) = -2122. \text{ fps}$ 
 $v_{mx}^{I}(0) = 0. \text{ fps}$ 
 $v_{mx}^{I}(0) = 0. \text{ fps}$ 
 $v_{mx}^{I}(0) = 0. \text{ fps}$ 
 $v_{mx}^{I}(0) = 1.5 \text{ seconds}$ 
 $v_{mx}^{I}(0) = 29.197 \text{ slugs/ft}^{2}$ 

$$\underline{\mathbf{R}} = \begin{bmatrix} \overline{\mathbf{3}} \cdot \mathbf{E} - 5 & 0 & 0 \\ 0 & 500 & 0 \\ 0 & 0 & 100 \end{bmatrix}$$

These plots were generated by estimating  $\tau_{\mathbf{f}}$  with the first order filter.  $\tau_{\mathbf{f}}$  was initialized at 1.5 seconds with its truth model value defined as 0.85 seconds. The truth model value was determined from an interative search as described in Chapter IV.

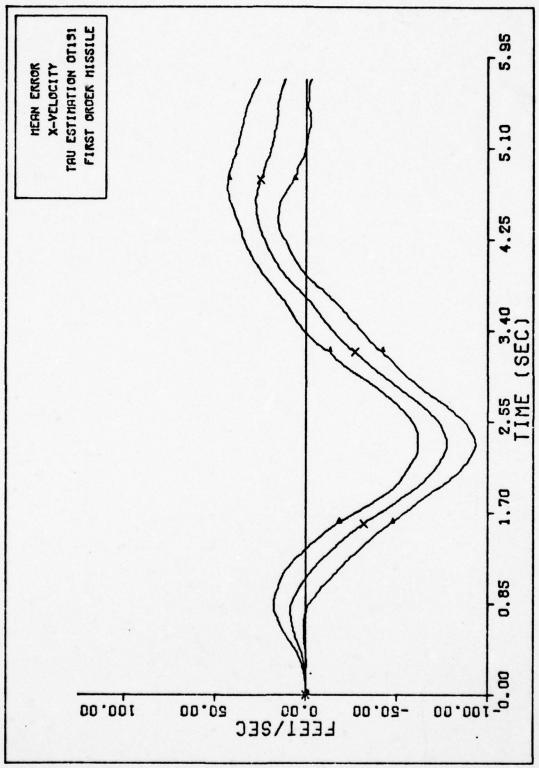


Fig. 179. X-VELOCITY FIRST ORDER MISSILE

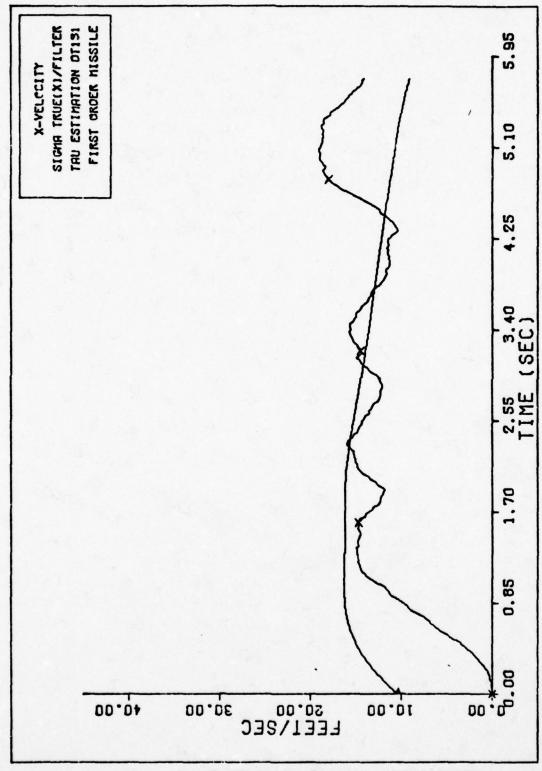


Fig. 180. X-VELOCITY SIGMAS FIRST ORDER

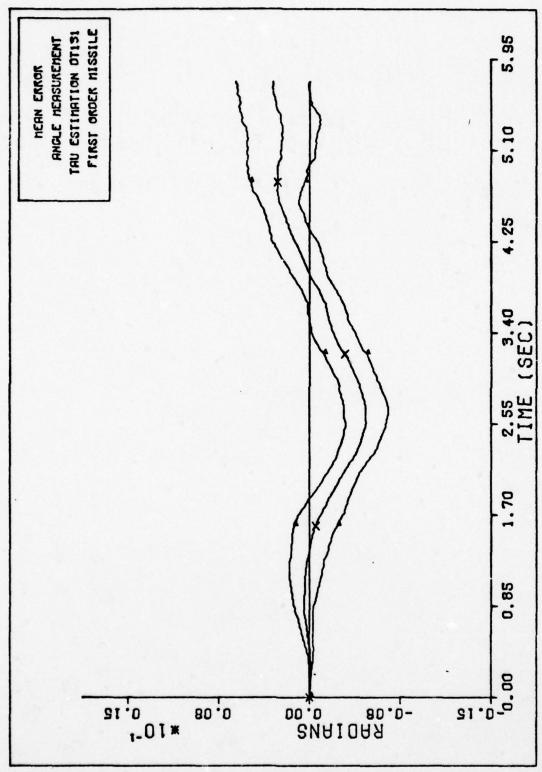


Fig. 181. ANGLE MEASUREMENT FIRST ORDER MISSILE

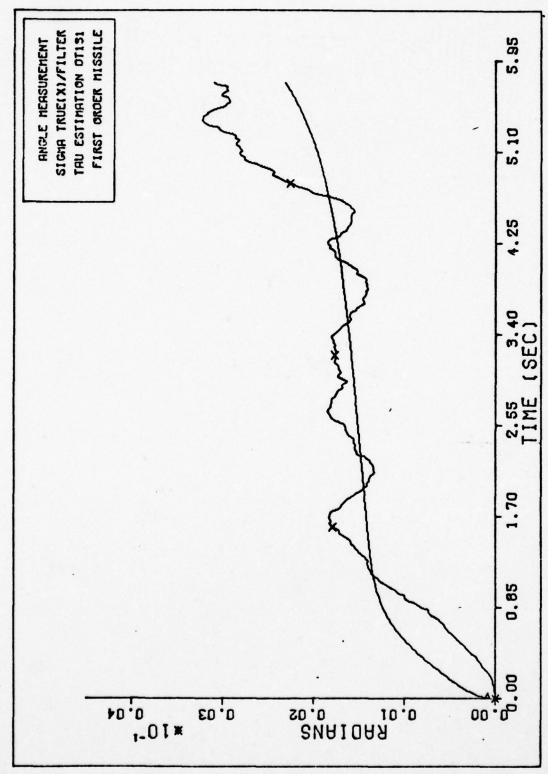


Fig. 182. ANGLE MERSUREMENT SIGMAS FIRST ORDER

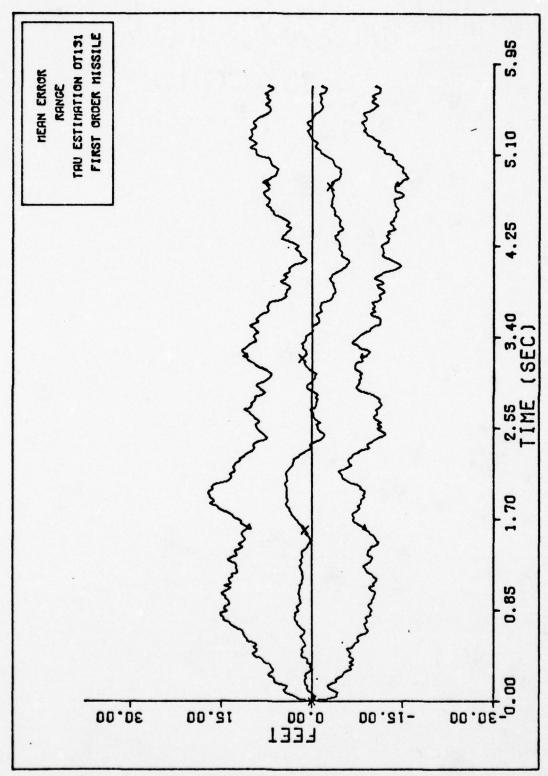


Fig. 183. RANGE FIRST ORDER MISSILE

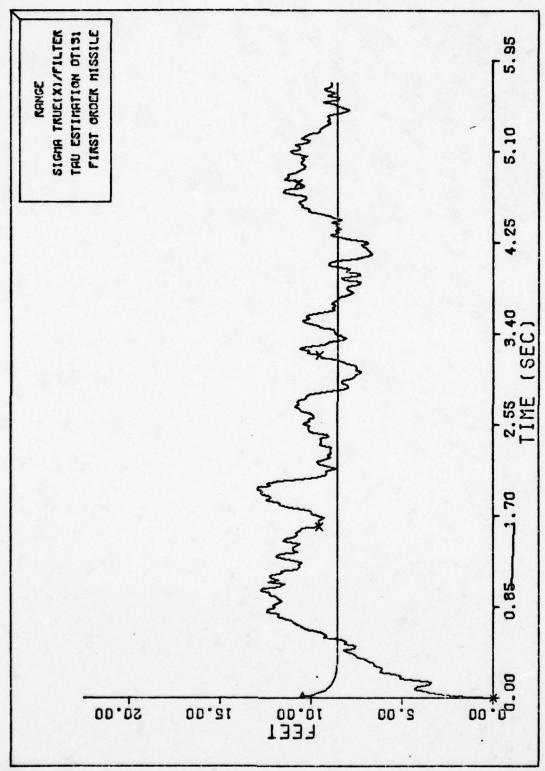


Fig. 184. RANGE SIGMAS FIRST ORDER

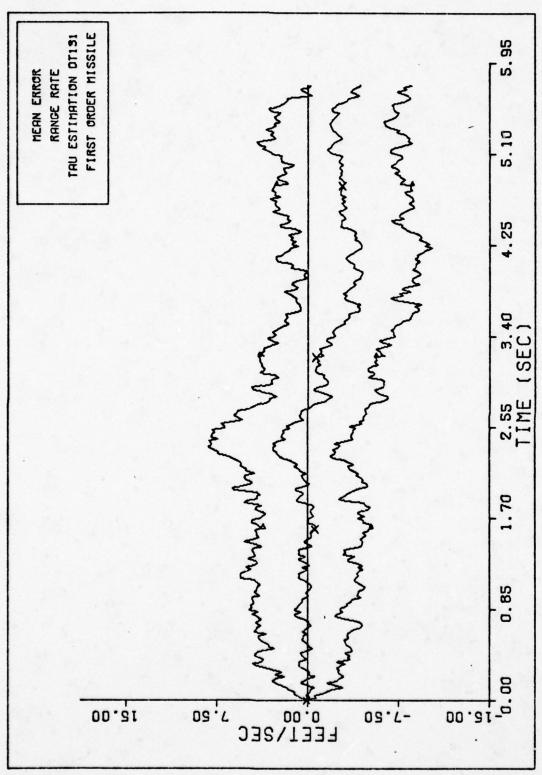


Fig. 185. RANGE RATE FIRST ORDER MISSILE

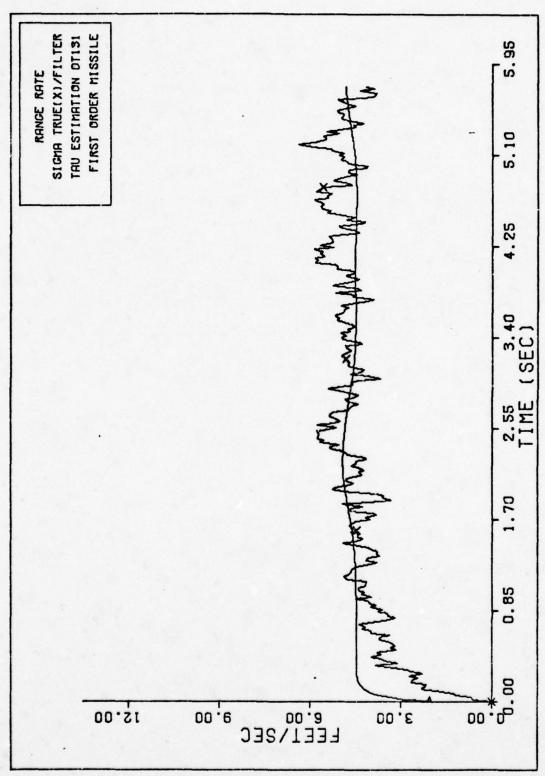


Fig. 186. RANGE RATE SIGMAS FIRST ORDER

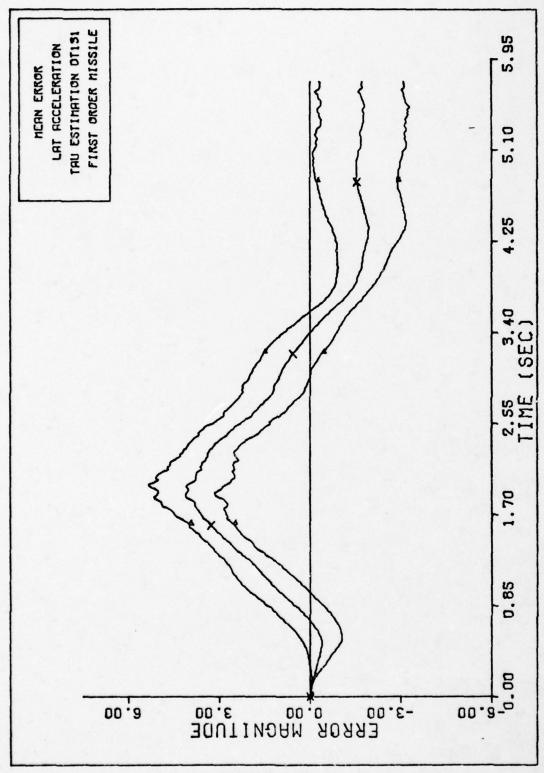


Fig. 187. LAT ACCELERATION FIRST ORDER MISSILE

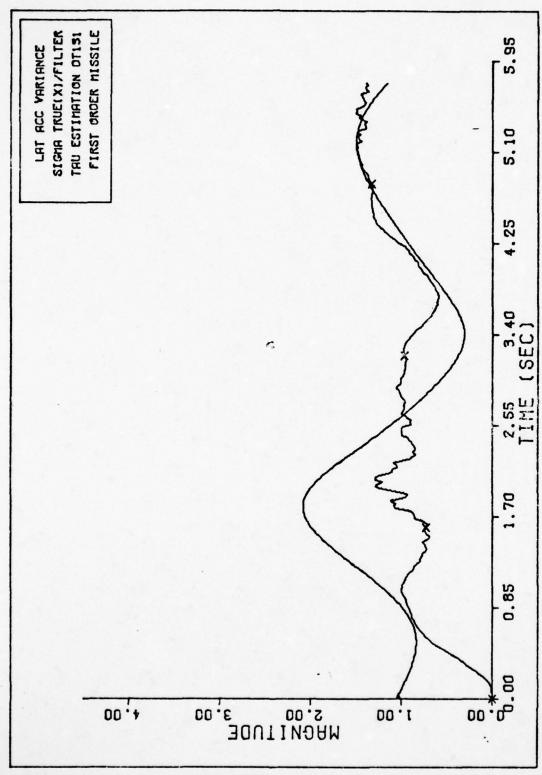


Fig. 188. LAT ACCELERATION SIGMAS FIRST ORDER

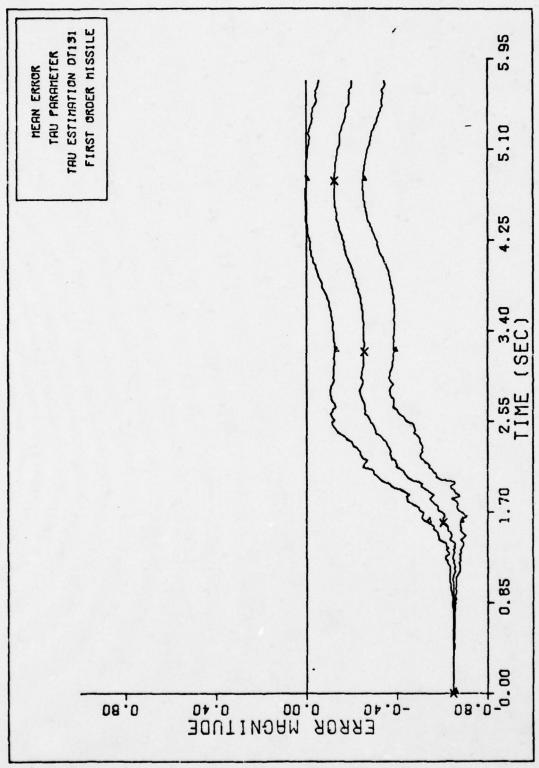


Fig. 189. TAU PARAMETER FIRST ORDER MISSILE

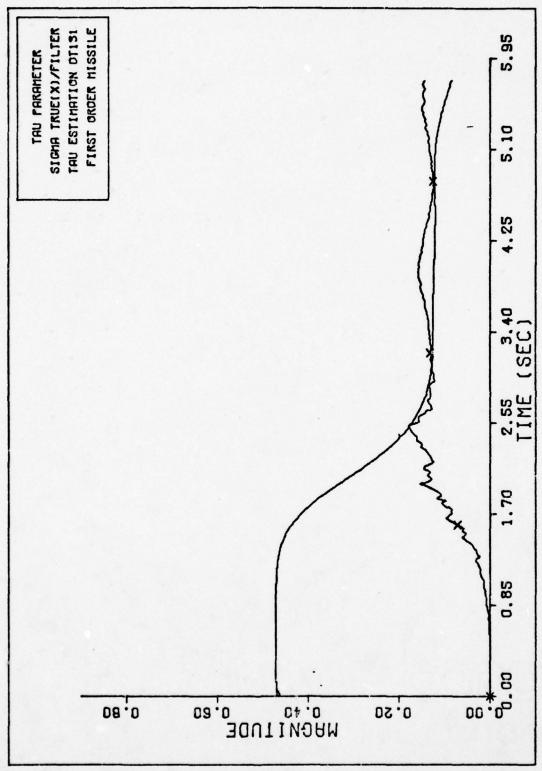


Fig. 190. TRU PARAMETER SIGMAS FIRST ORDER

## If Estimation - If Initialized at .3

The initial state estimates and the tuning parameters for this case are

$$v_{mx}^{I}(0) = 1225.7 \text{ fps}$$
 $e(0) = 4.363345 \text{ radians}$ 
 $R(0) = 10000. \text{ feet}$ 
 $\dot{R}(0) = -2122. \text{ fps}$ 
 $a_{L}(0) = 0.$ 
 $n(0) = 4.5$ 
 $\tau_{f}(0) = .3 \text{ seconds}$ 

 $M/S(0) = 29.197 \text{ slugs/ft}^2$ 

$$\underline{\mathbf{R}} = \begin{bmatrix} \overline{\mathbf{3.E-5}} & 0. & 0. \\ 0. & 500. & 0. \\ 0. & 0. & 100. \end{bmatrix}$$

These plots were generated by estimating  $\tau_{\mathbf{f}}$  with the first order filter.  $\tau_{\mathbf{f}}$  was initialized at 0.3 seconds in the filter with its truth model value defined as 0.85 seconds. The truth model value was determined from an iterative search as described in Chapter IV.

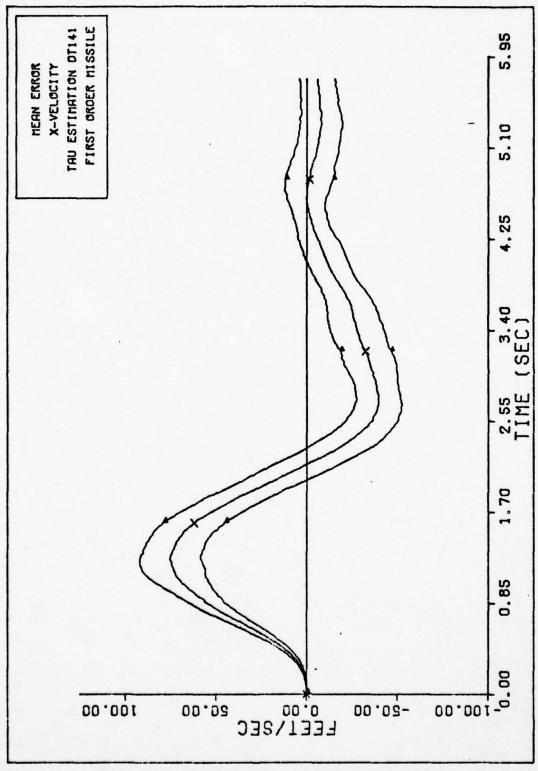


Fig. 191. X-VELOCITY FIRST ORDER MISSILE

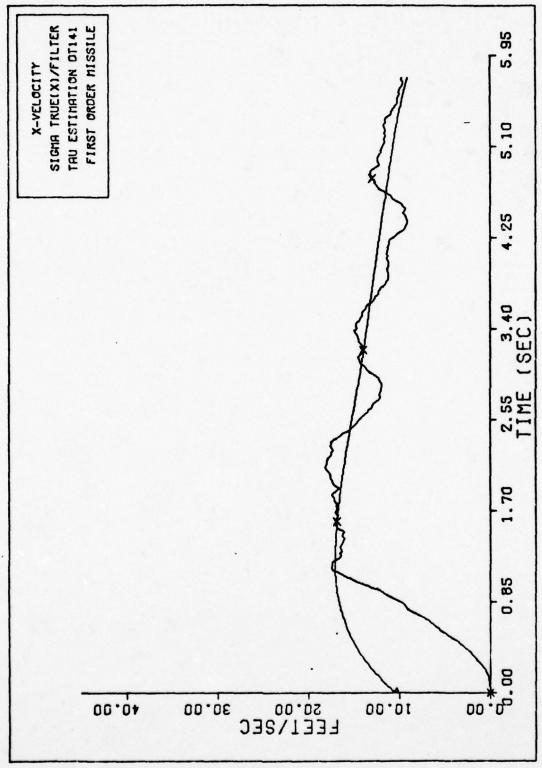


Fig. 192. X-VELOCITY SIGMAS FIRST ORDER

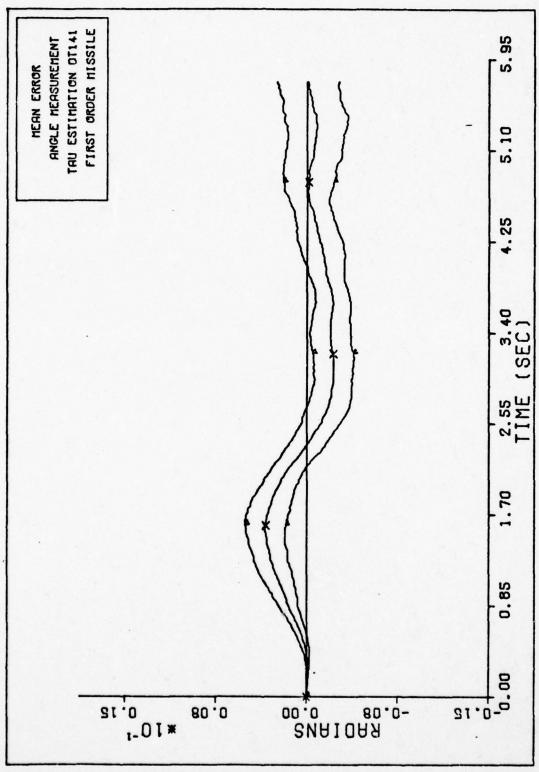


Fig. 193. ANGLE MEASUREMENT FIRST ORDER MISSILE

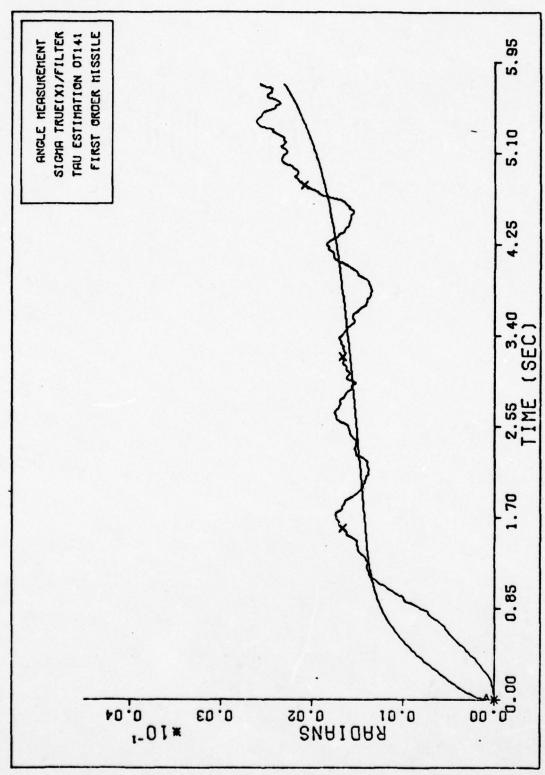


Fig. 194. ANGLE MEASUREMENT SIGMAS FIRST ORDER

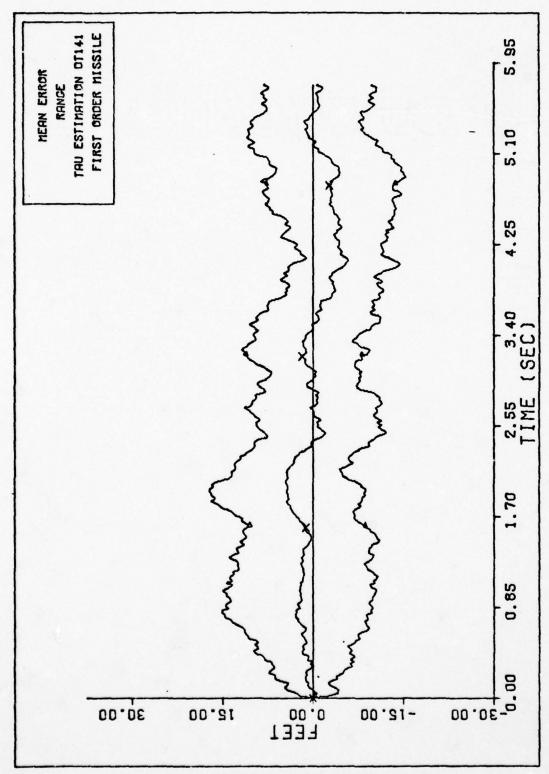


Fig. 195. RANGE FIRST ORDER MISSILE

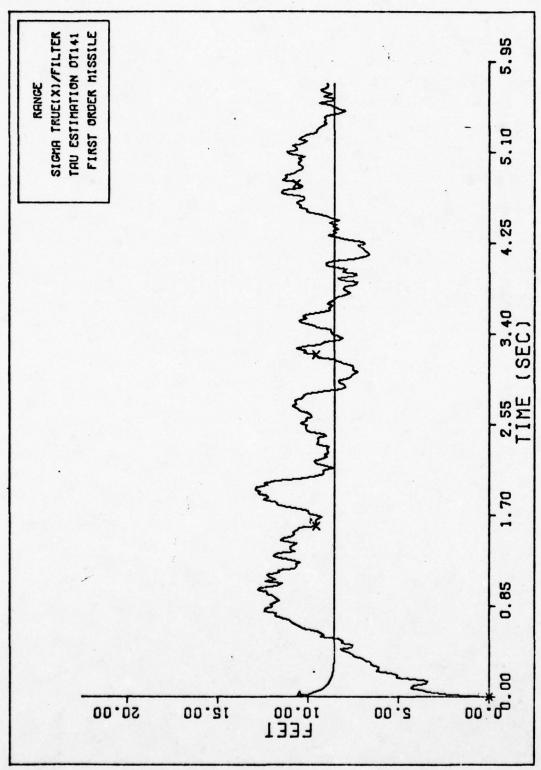


Fig. 196. RANGE SIGMAS FIRST ORDER

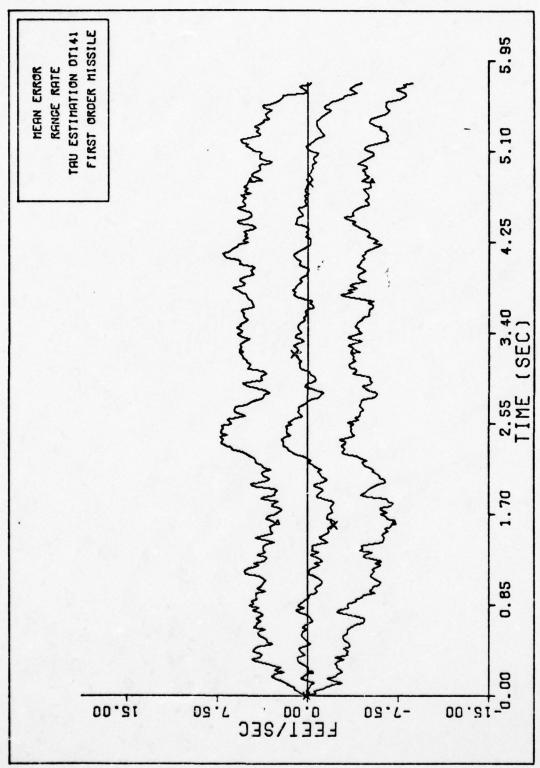


Fig. 197. RANGE RATE FIRST ORDER MISSILE

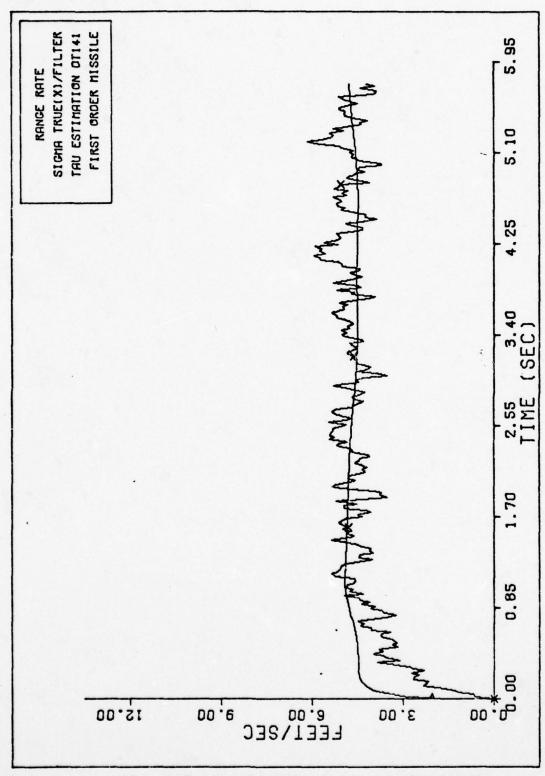


Fig. 198. RANGE RATE SIGMAS FIRST ORDER

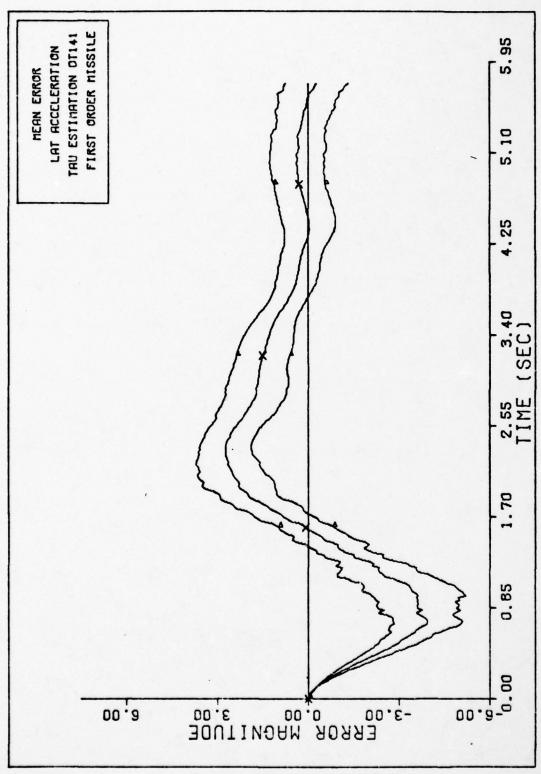


Fig. 199. LAT ACCELERATION FIRST ORDER MISSILE

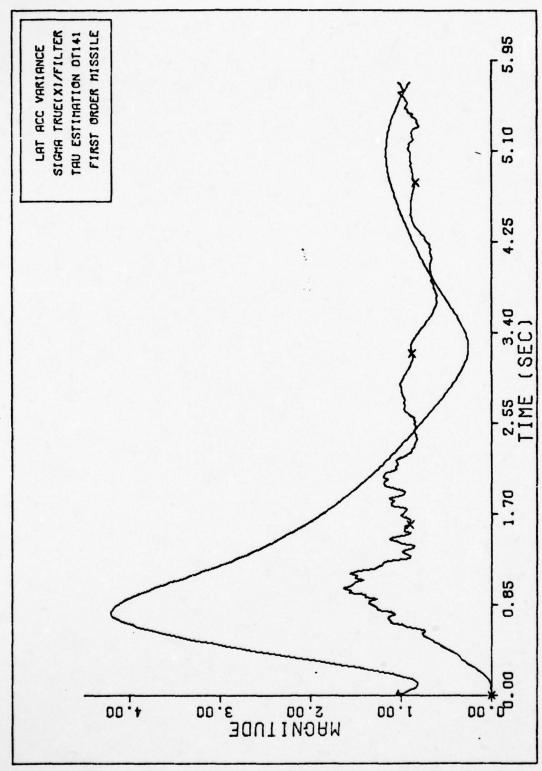


Fig. 200. LAT ACCELERATION SIGMAS FIRST ORDER

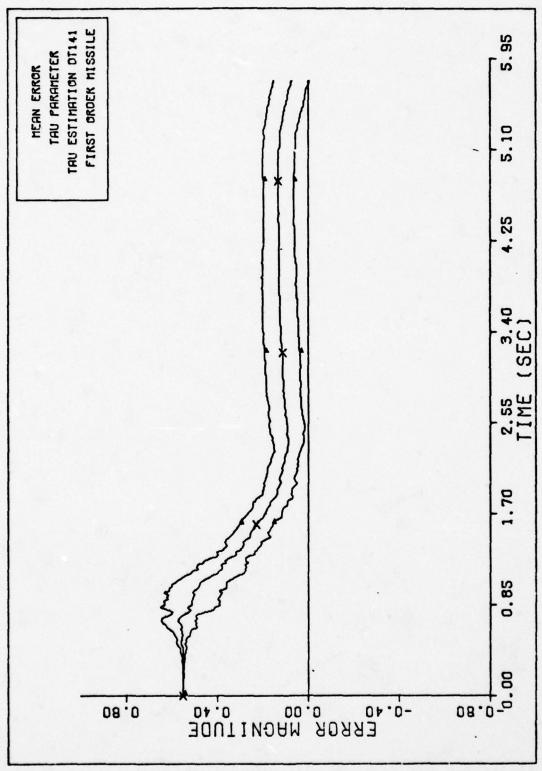


Fig. 201. TAU PARAMETER FIRST ORDER MISSILE

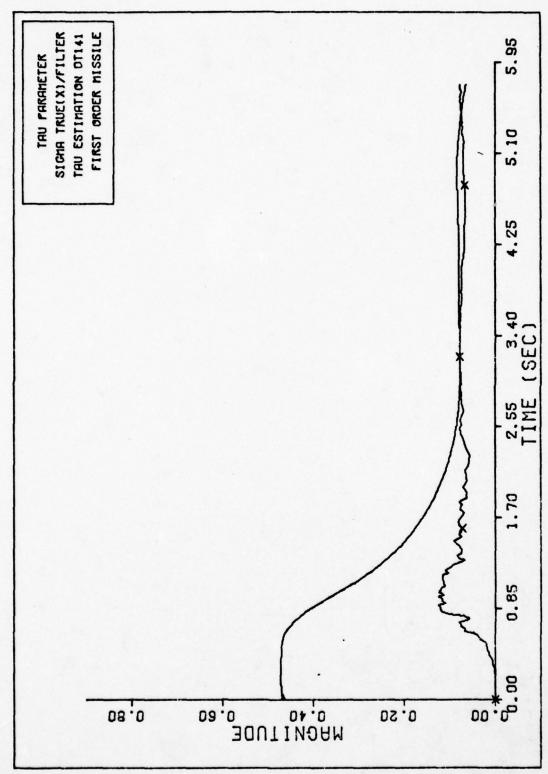


Fig. 202. TAU PARAMETER SIGMAS FIRST ORDER

## M/S Estimation - M/S Initialized at 45.

The initial state estimates and the tuning parameters for this case are

$$V_{mx}^{I}(0) = 1225.7 \text{ fps}$$
 $\dot{\theta}(0) = 4.363345 \text{ radians}$ 
 $R(0) = 10000. \text{ feet}$ 
 $\dot{R}(0) = -2122. \text{ fps}$ 
 $a_{L}(0) = 0.$ 
 $n(0) = 4.5$ 
 $\tau_{f}(0) = .85 \text{ seconds}$ 
 $M/S(0) = 45. \text{ slugs/ft}^{2}$ 

$$\underline{\mathbf{R}} = \begin{bmatrix} \overline{\mathbf{3}}.\mathbf{E} - 5 & 0. & 0. \\ 0. & 500. & 0. \\ 0. & 0. & 100. \end{bmatrix}$$

These plots were generated by the first order filter when estimating the M/S ratio. The initial value of this ratio set in the filter was 45.0 slugs/ft<sup>2</sup>. The true value was set at 29.197 slugs/ft<sup>2</sup>. This parameter along with the five dynamic states of the missile model were the only estimates of the first order filter in this case.

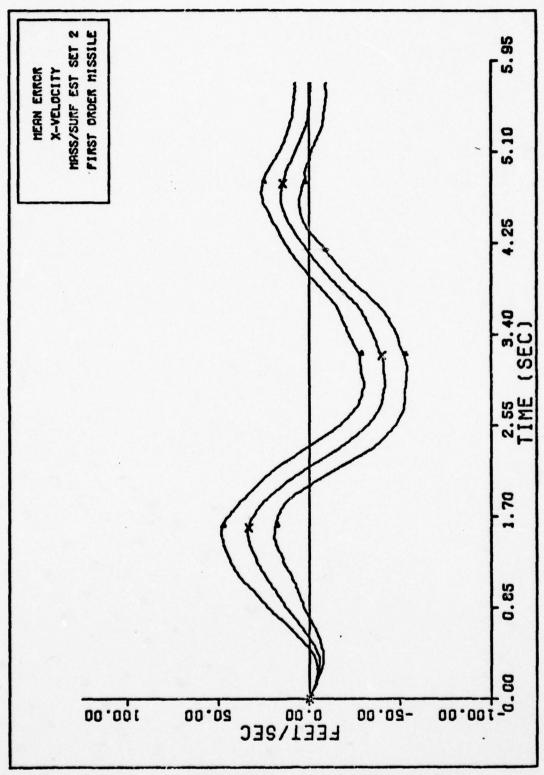


Fig. 203. X-VELOCITY FIRST ORDER MISSILE

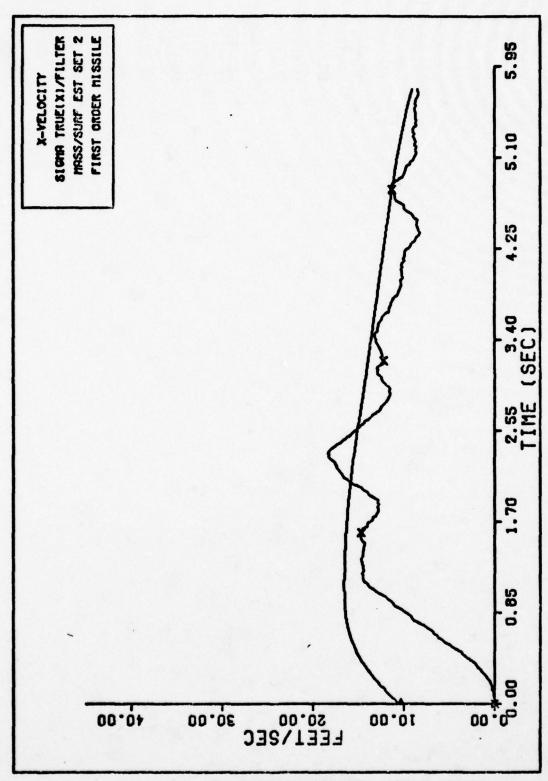


Fig. 204. X-VELOCITY SIGMAS FIRST ORDER

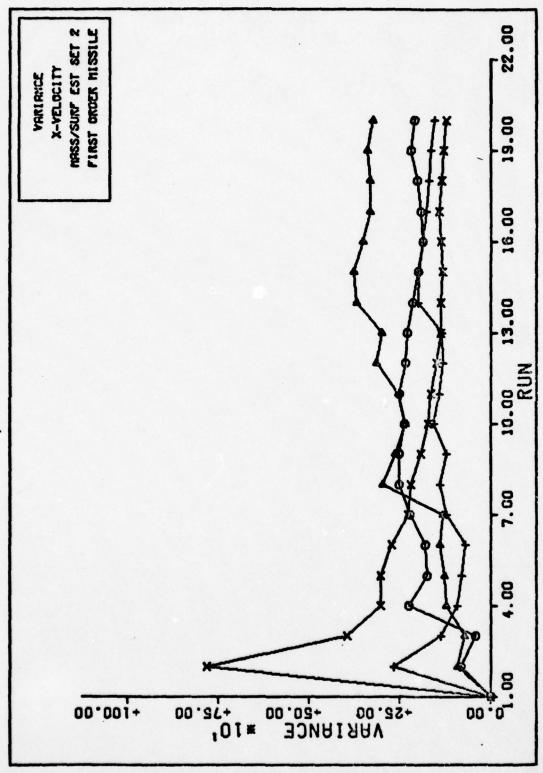


Fig. 205. VARIANCE CONVERGENCE

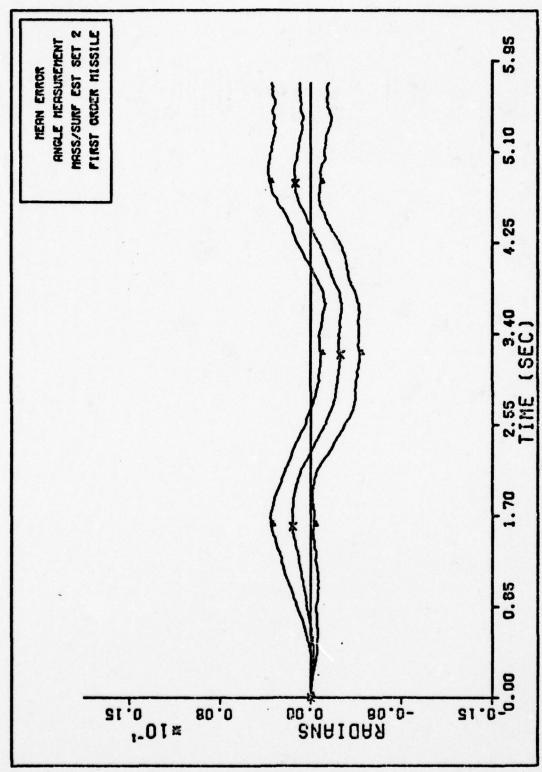


Fig. 206. ANGLE MEASUREMENT FIRST ORDER MISSILE

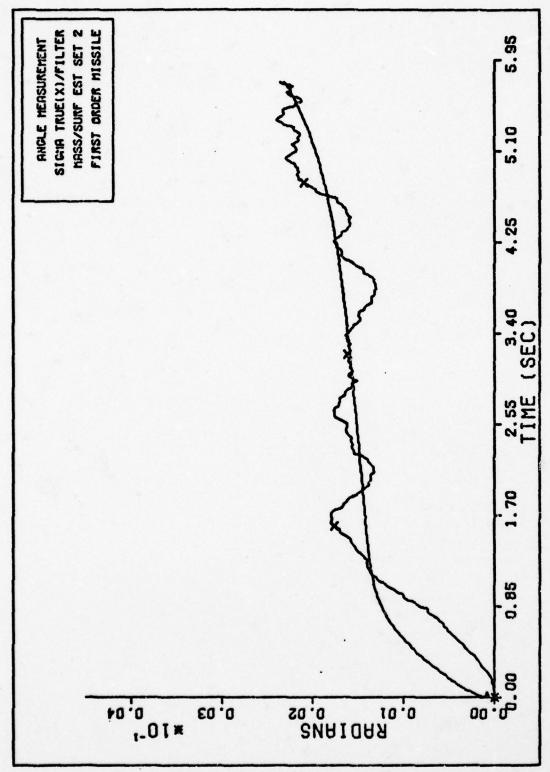


Fig. 207. ANDLE MEASUREMENT SIGMAS FIRST ORDER

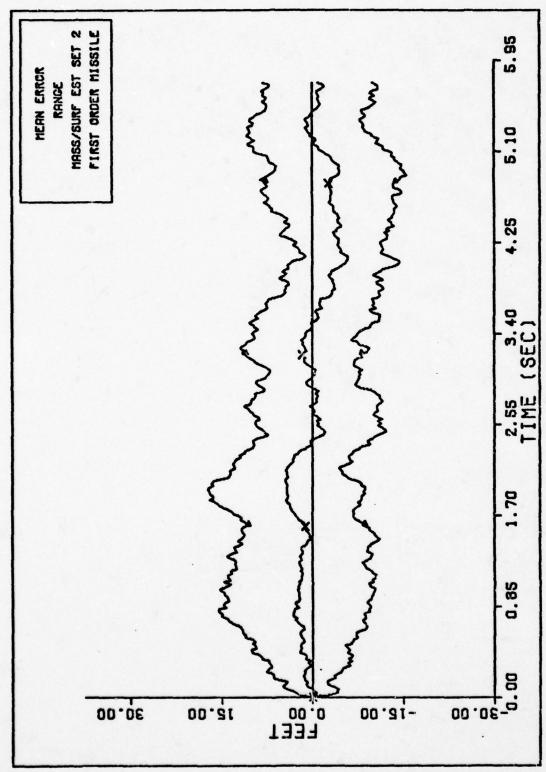


Fig. 208. RANGE FIRST ORDER MISSILE

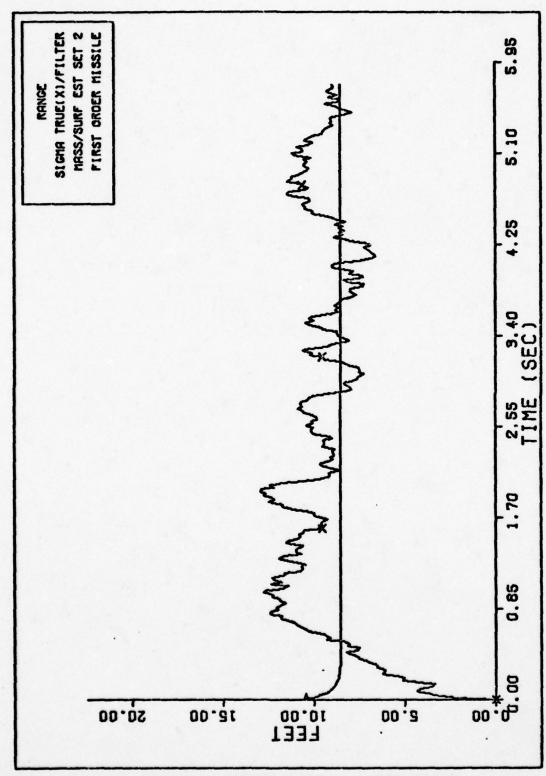


Fig. 209. RANGE SIGMAS FIRST ORDER

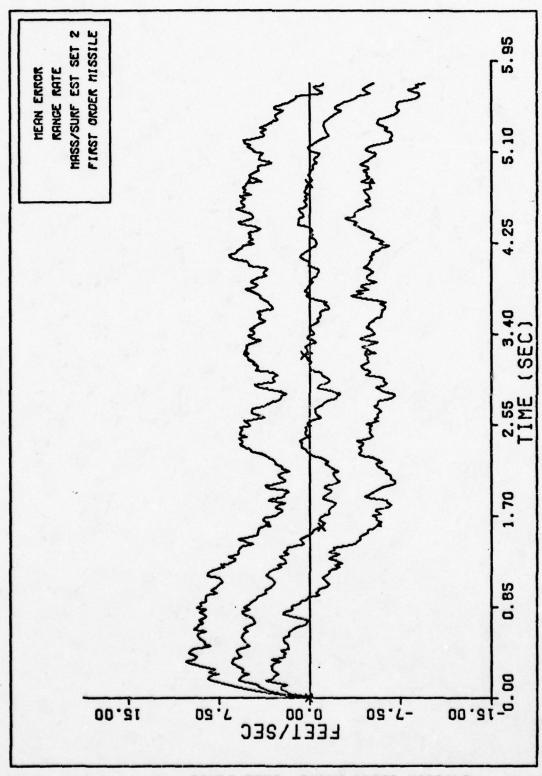


Fig. 210. RANGE RATE FIRST ORDER MISSILE

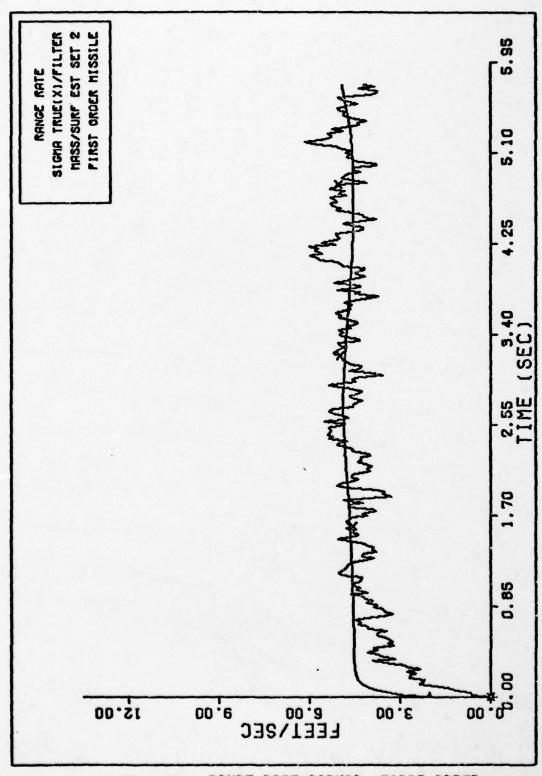


Fig. 211. RANGE RATE SIGMAS FIRST DRDER

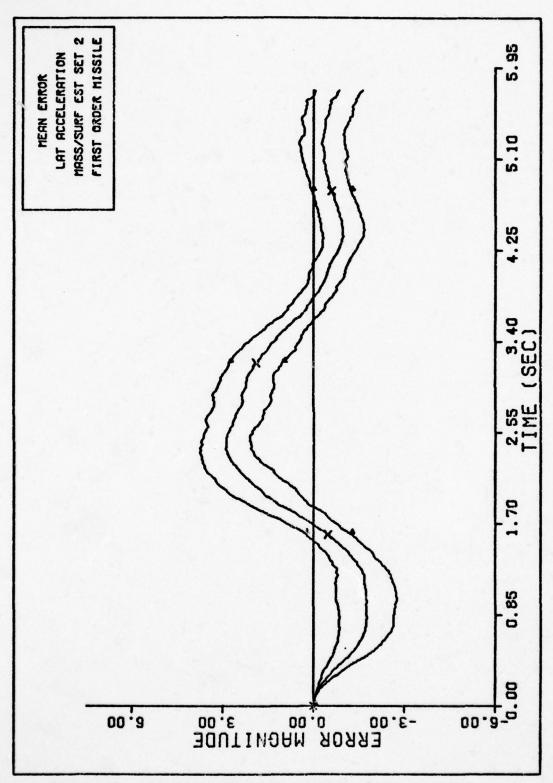


Fig. 212. LAT ACCELERATION FIRST ORDER MISSILE

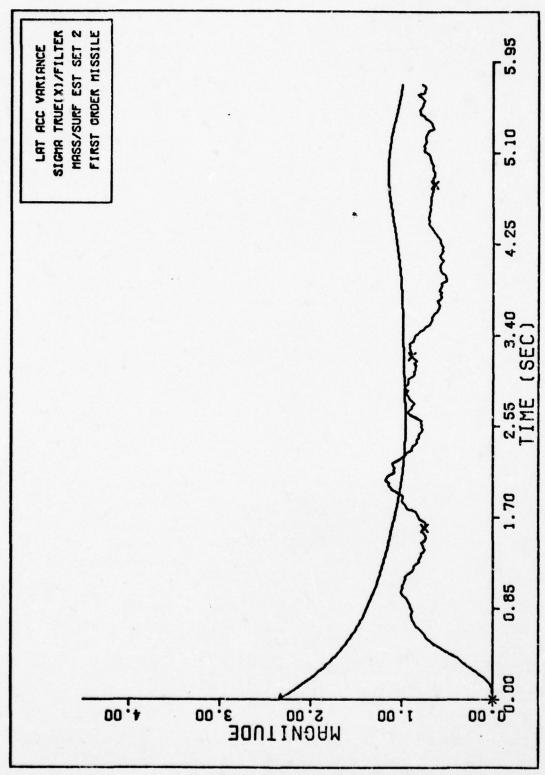


Fig. 213. LAT ACCELERATION SIGMAS FIRST ORDER

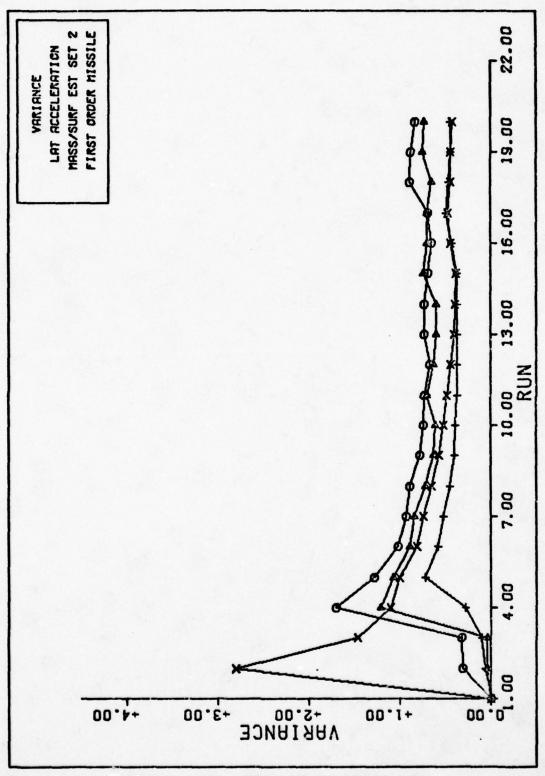


Fig. 214. VARIANCE CONVERGENCE

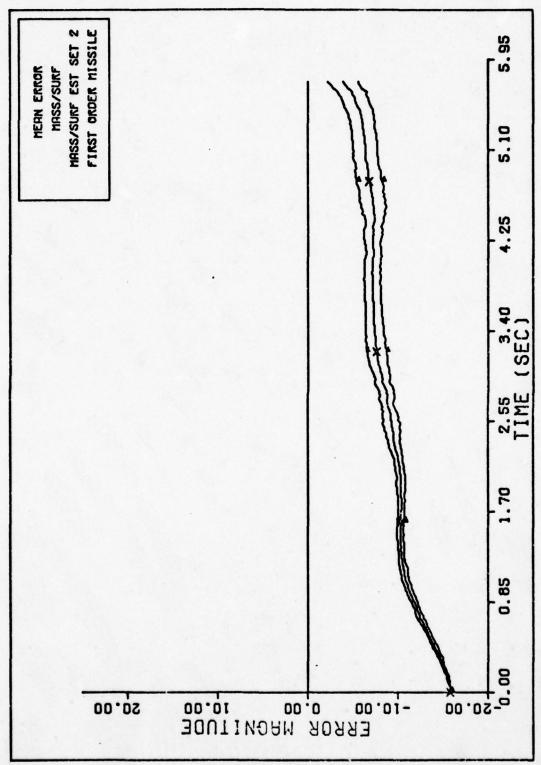


Fig. 215. MASS/SURF FIRST CROER MISSILE

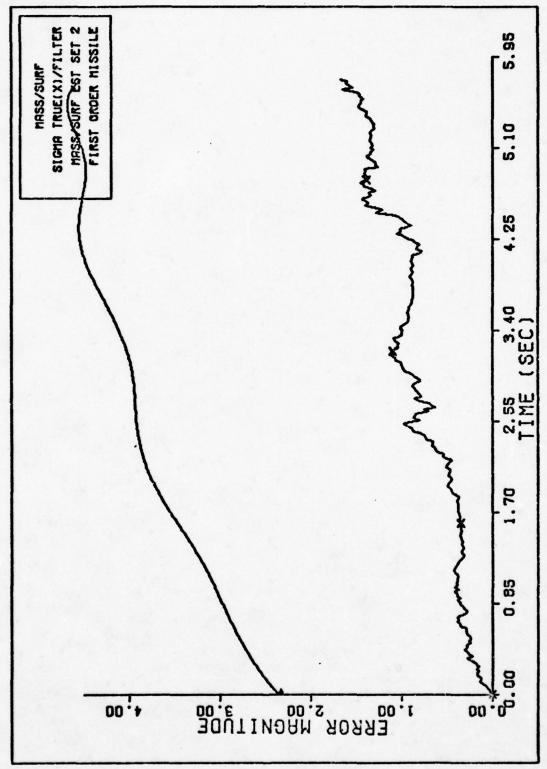


Fig. 216. MASS/SURF SIGMAS FIRST ORDER

## M/S Estimation - M/S Initialized at 15.

The initial state estimates and the tuning parameters for this case are

$$v_{mx}^{I}(0) = 1225.7 \text{ fps}$$

$$V_{mx}^{I}(0) = 1225.7 \text{ fps}$$
  
 $\dot{\Theta}(0) = 4.363345 \text{ radians}$ 

$$R(0) = 10000.$$
 feet

$$\dot{R}(0) = -2122. \text{ fps}$$

$$a_{L}(0) = 0.$$

$$n(0) = 4.5$$

$$\tau_{\mathbf{f}}(0) = .85 \text{ seconds}$$

$$M/S(0) = 15 \text{ slugs/ft}^2$$

$$\underline{\mathbf{R}} = \begin{bmatrix} \overline{\mathbf{3.E-5}} & 0. & 0. \\ 0. & 500. & 0. \\ 0. & 0. & 100. \end{bmatrix}$$

These plots were generated by the first order filter when estimating the M/S ratio. The initial value of this ratio set in the filter was 15.0 slugs/ft<sup>2</sup>. The true value was set at 29.197 slugs/ft<sup>2</sup>. This parameter along with the five dynamic states of the missile model were the only estimates of the first order filter in this case.

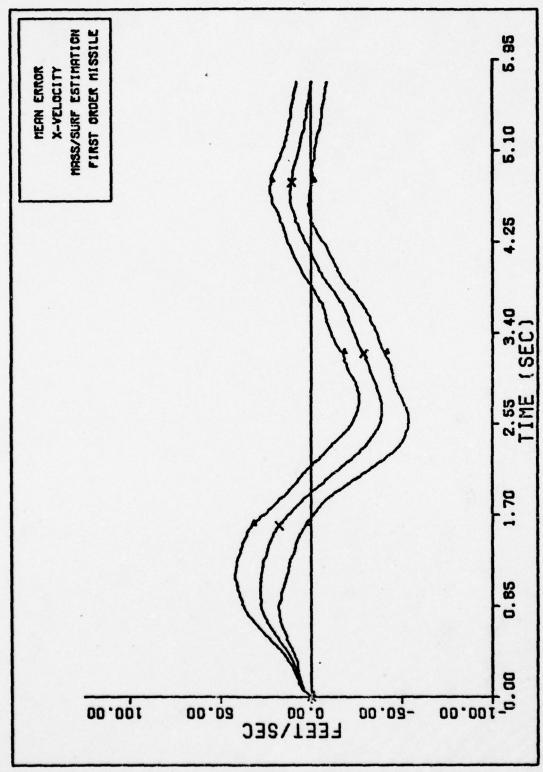


Fig. 217. X-VELOCITY FIRST ORDER MISSILE

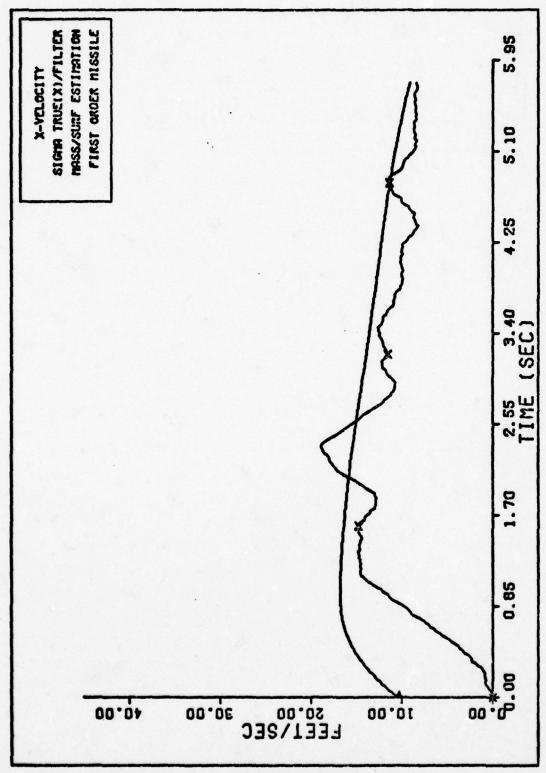


Fig. 218. X-VELOCITY SIGMAS FIRST ORDER

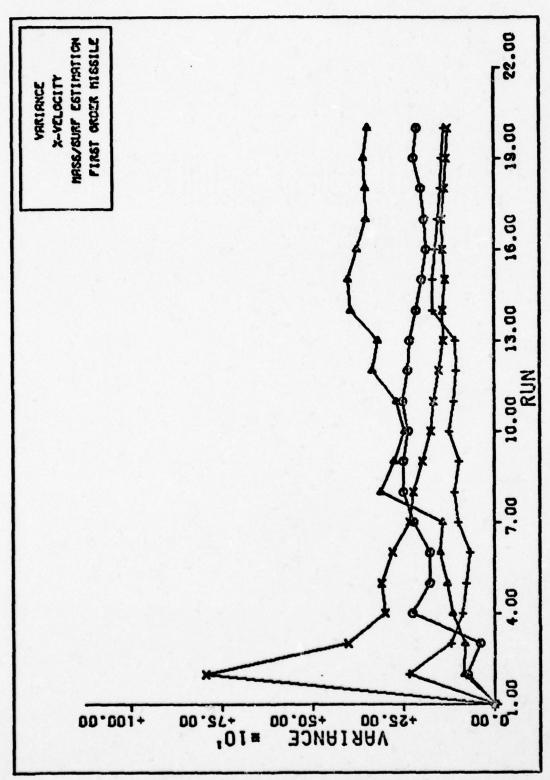


Fig. 219. VARIANCE CONVERGENCE

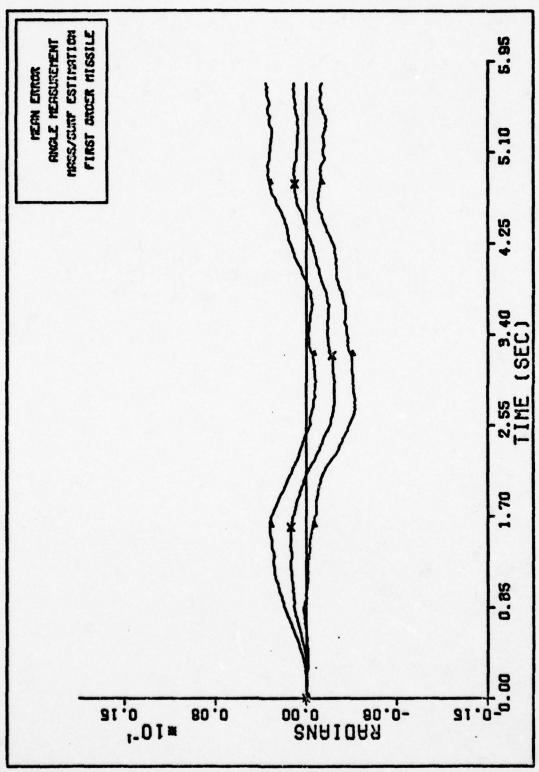


Fig. 220. ANGLE MERSUREMENT FIRST DRDER MISSILE

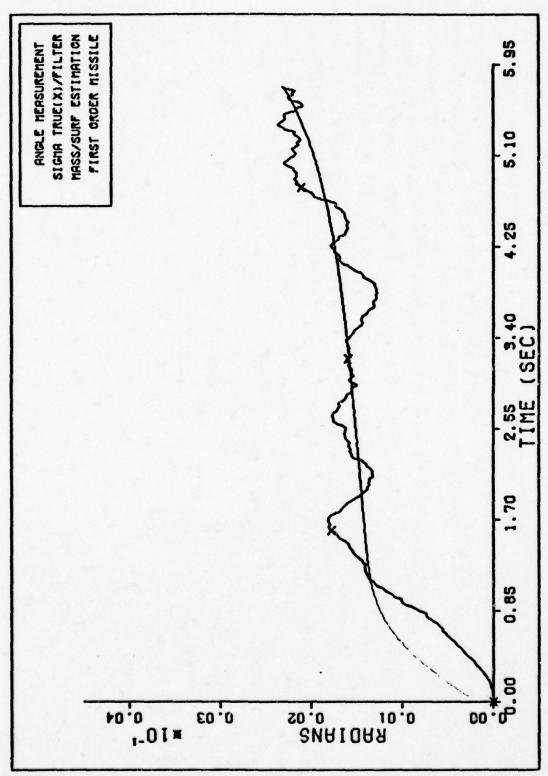


Fig. 221. ANGLE MEASUREMENT SIGMAS FIRST ORDER

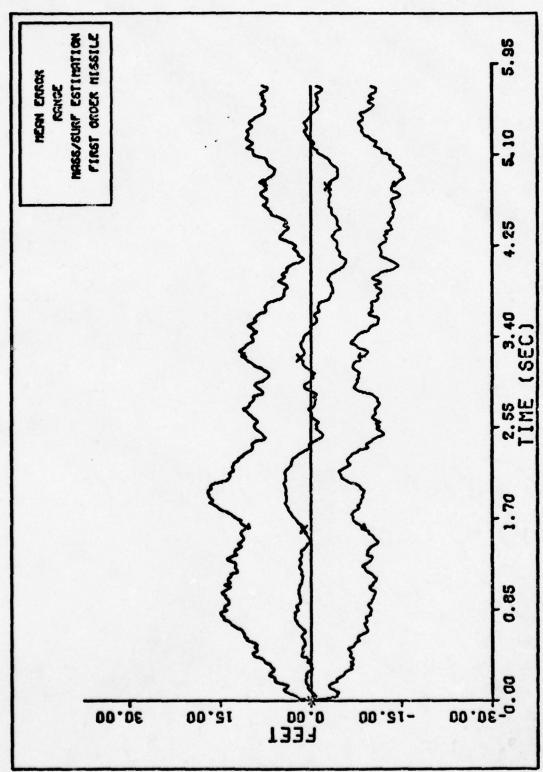


Fig. 222. RANGE FIRST ORDER MISSILE

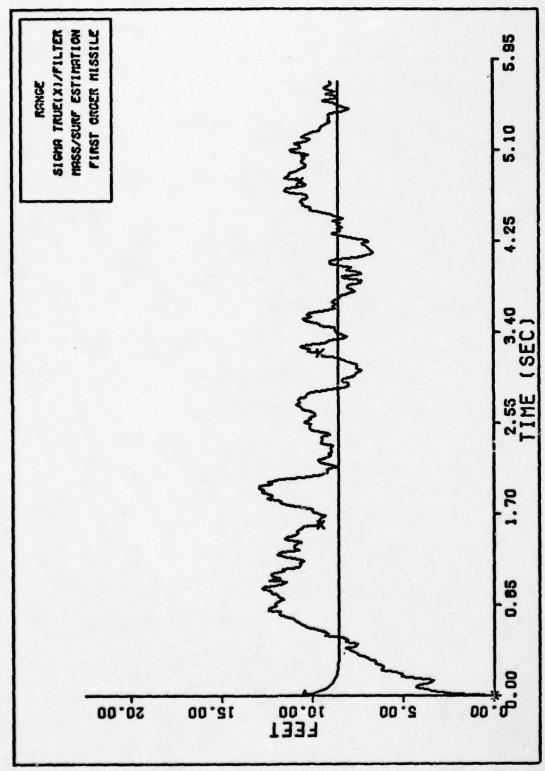


Fig. 223. RANGE SIGMAS FIRST ORDER

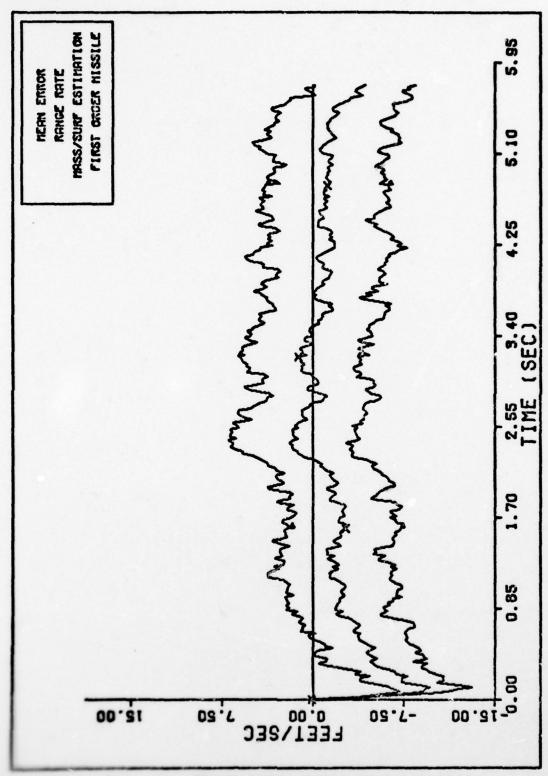


Fig. 224. RANGE RATE FIRST ORDER MISSILE

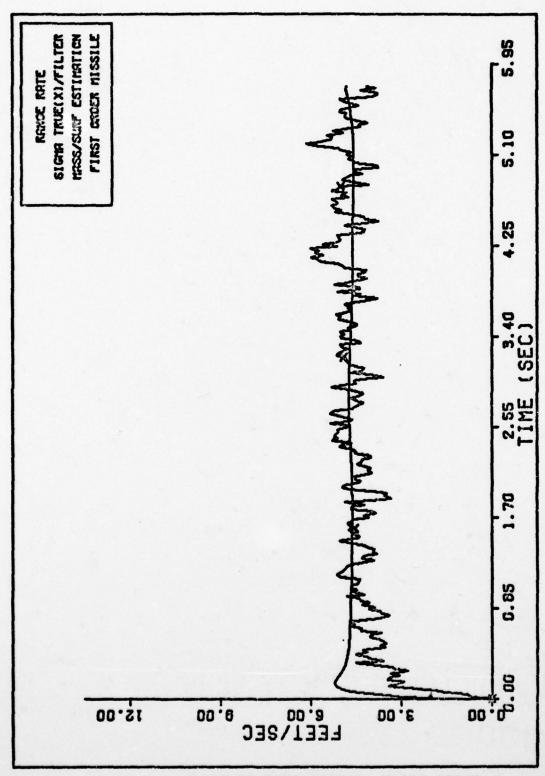


Fig. 225. RANGE RATE SIGMAS FIRST ORDER

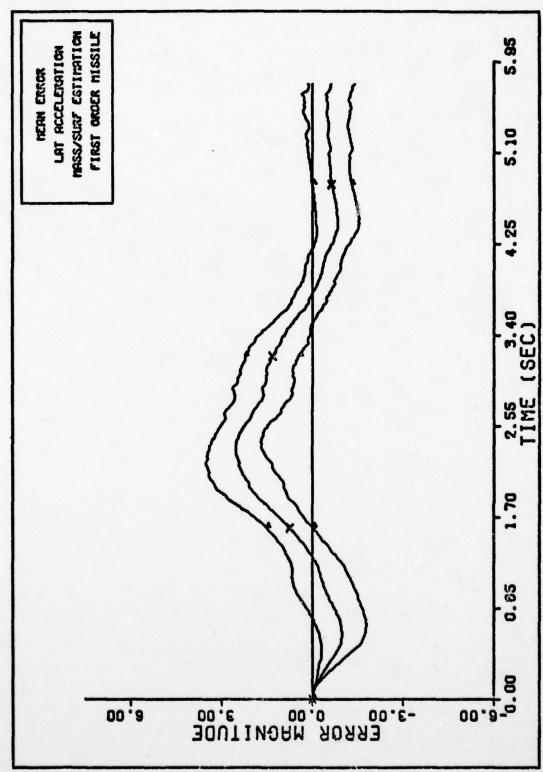


Fig. 226. LAT ACCELERATION FIRST CROER MISSILE

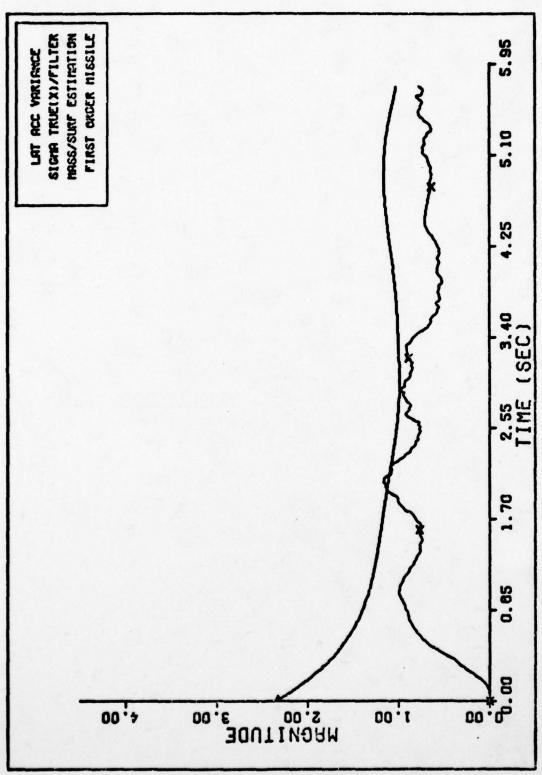


Fig. 227. LAT ACCELERATION SIGMAS FIRST ORDER

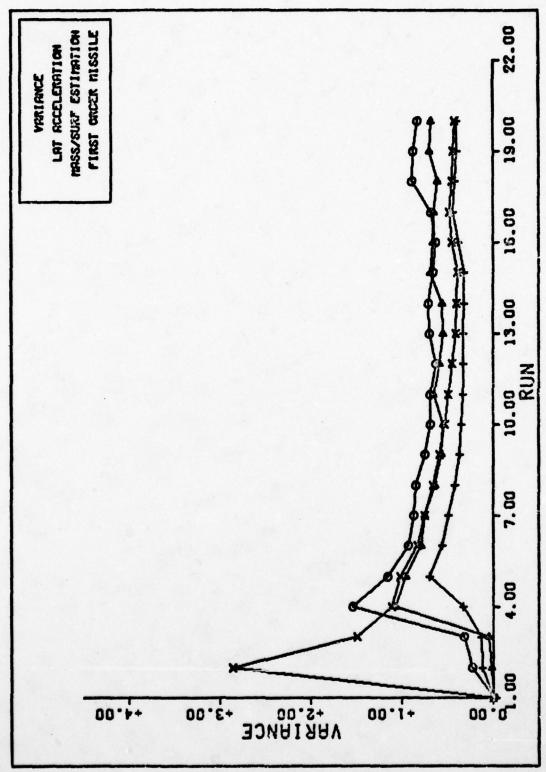


Fig. 228. VARIANCE CONVERGENCE

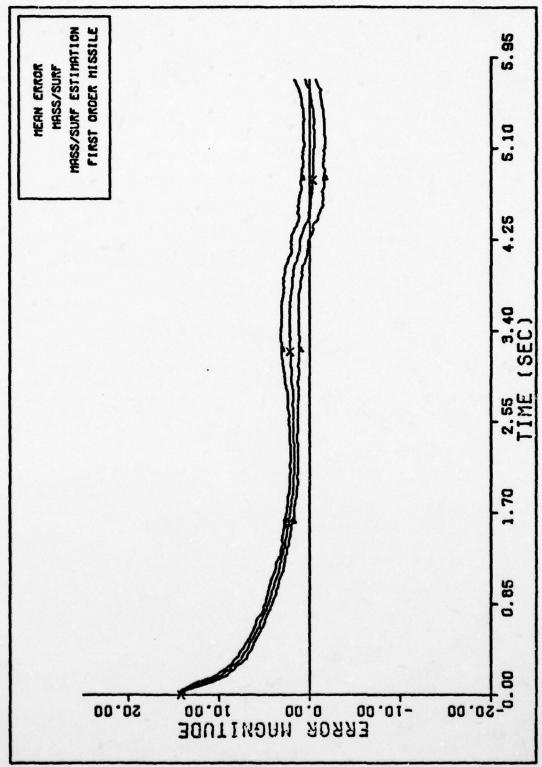


Fig. 229. MASS/SURF FIRST ORDER MISSILE

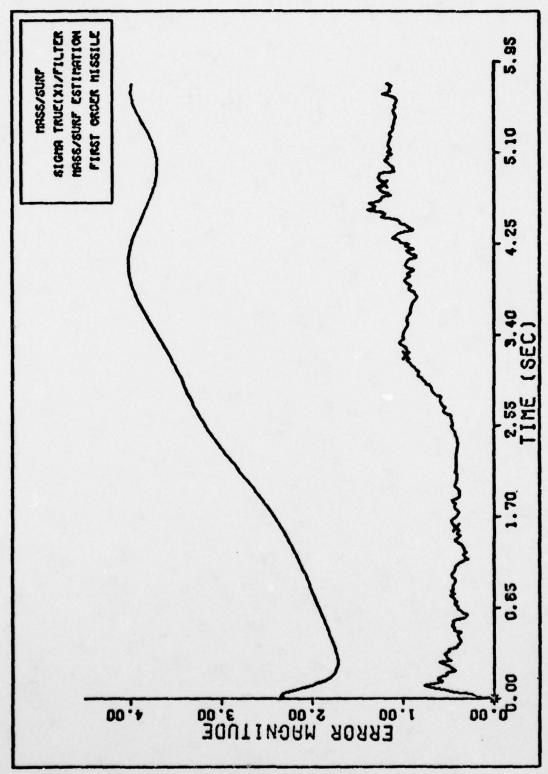


Fig. 230. MASS/SURF SIGMAS FIRST ORDER

## n and Tf Estimation (high-g scenario)

The initial state estimates and the tuning parameters for this case are

$$v_{mx}^{I}(0) = 1225.7 \text{ fps}$$
 $e(0) = 4.363345 \text{ radians}$ 
 $R(0) = 10000. \text{ feet}$ 
 $\dot{R}(0) = -2122. \text{ fps}$ 
 $a_{L}(0) = 0.$ 
 $n(0) = 6.$ 
 $\tau_{f}(0) = .3 \text{ seconds}$ 
 $M/S(0) = 29.197 \text{ slugs/ft}^{2}$ 

$$\underline{\mathbf{R}} = \begin{bmatrix} \overline{\mathbf{3}} \cdot \mathbf{E} - 5 & 0 & 0 \\ 0 & 500 & 0 \\ 0 & 0 & 100 \end{bmatrix}$$

250. 0. 0. 0. 0. 0. 0. 0. 1.E-6 0. 0. 0. 0. 0. 0. 0. 500. 0. 0. 0. 0. 0. 0. 0. 0.0. 0. 0.0. 0. 0. 200. 0. 0. 0. 0. 0. 10. 0. 0. 0. 0. 0. .01 0. 0. 0. 0. 0. .001 0. 0. 0. 0. 0. 0. 0. 0.

These results were produced when estimating n and  $\tau_{\rm f}$  together with the five dynamic states. Since these parameters only appeared together in the model of the guidance strip, they were considered the most difficult combination to estimate simultaneously. n was initialized at 6., and  $\tau_{\rm f}$  was initialized at 0.3 seconds. The true values of n and  $\tau_{\rm f}$  were 4.5 and 0.85 seconds respectively.

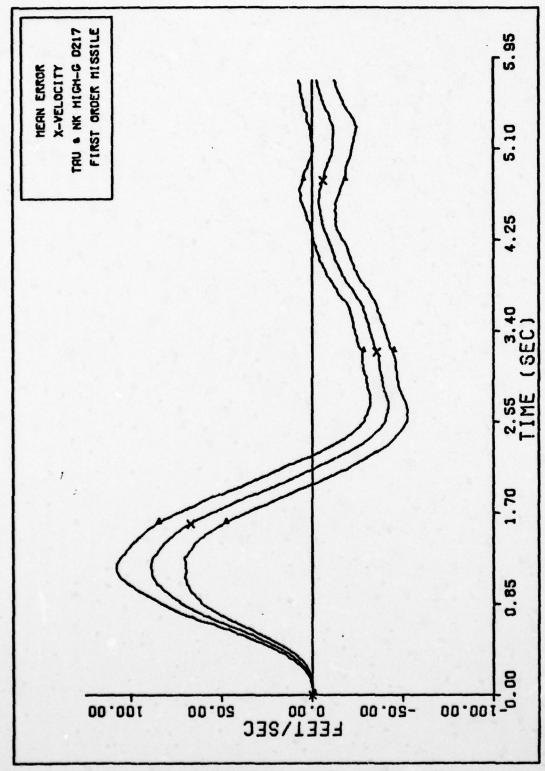


Fig. 231. X-VELOCITY FIRST ORDER MISSILE

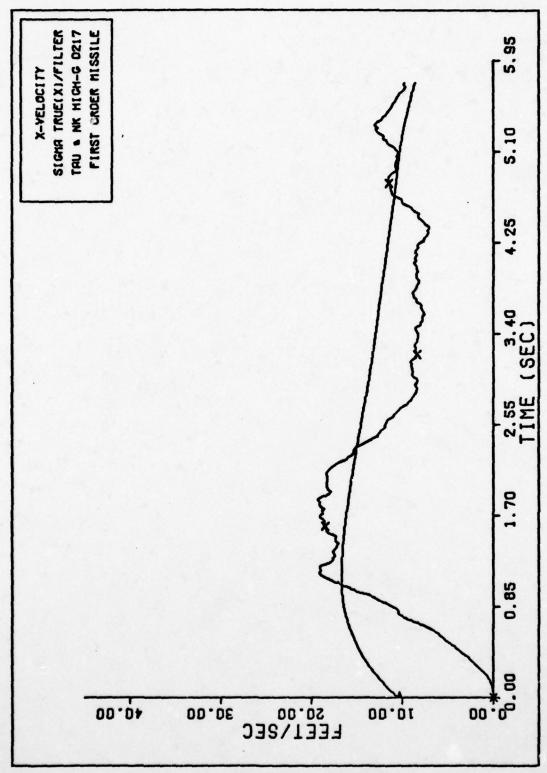


Fig. 232. X-VELOCITY SIGMAS FIRST ORDER

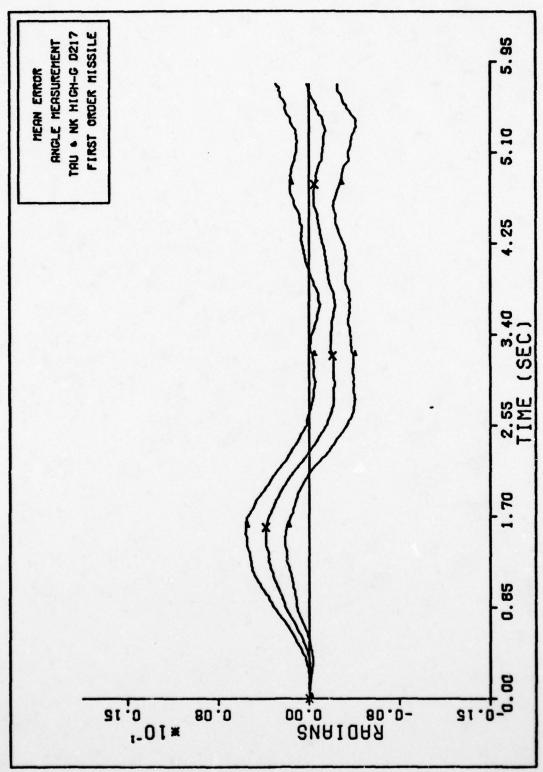


Fig. 233. ANGLE MEASUREMENT FIRST ORDER MISSILE

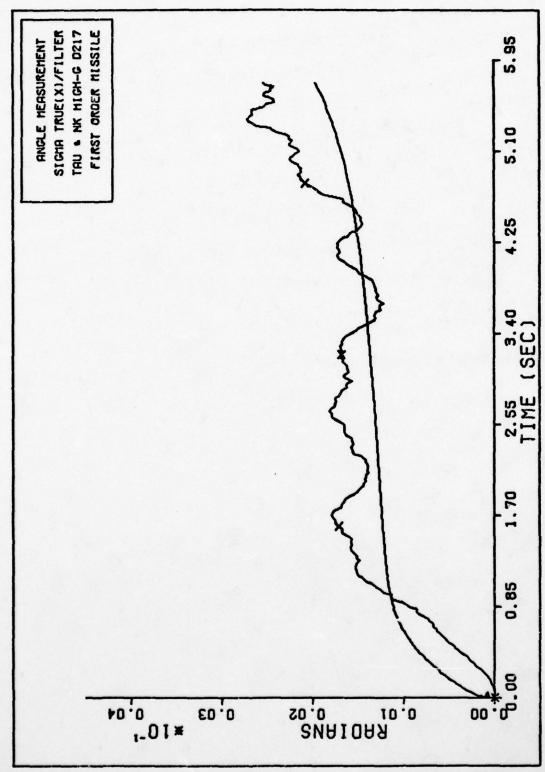


Fig. 234. ANGLE MEASUREMENT SIGMAS FIRST ORDER

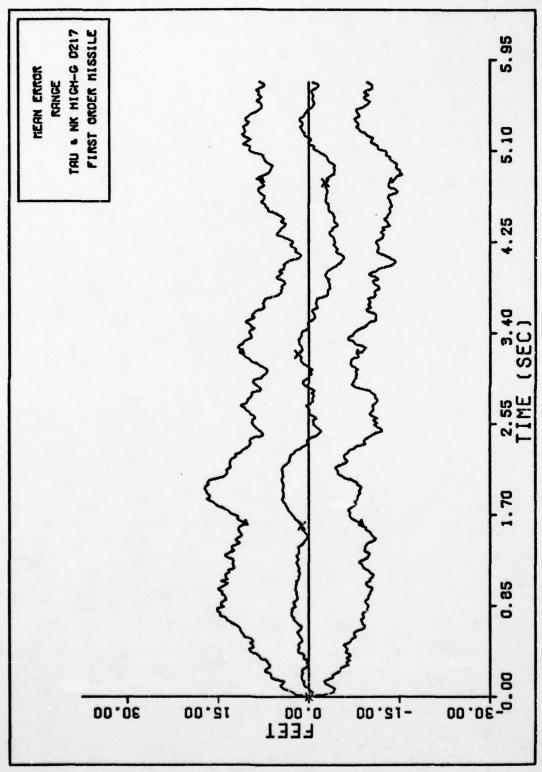
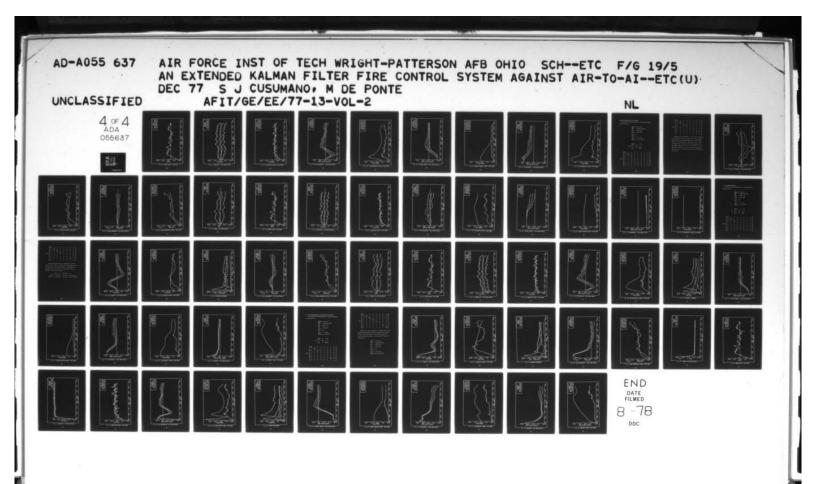


Fig. 235. RANGE FIRST ORDER MISSILE



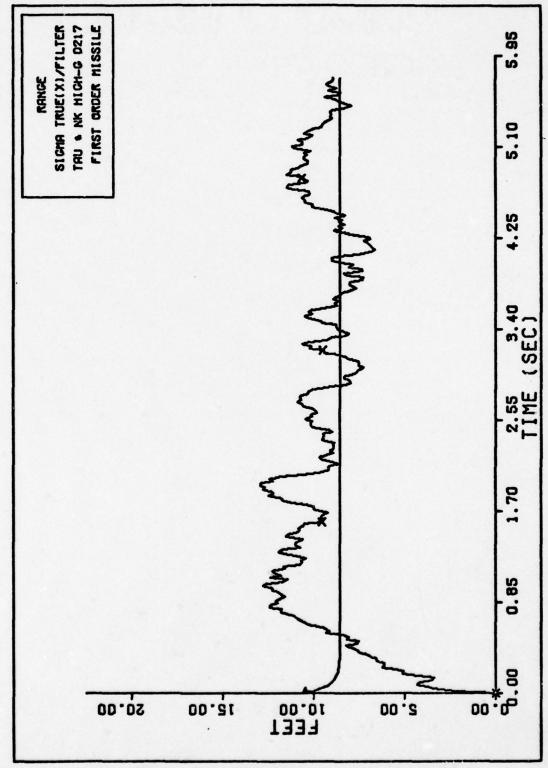


Fig. 236. RANGE SIGNAS FIRST GROER

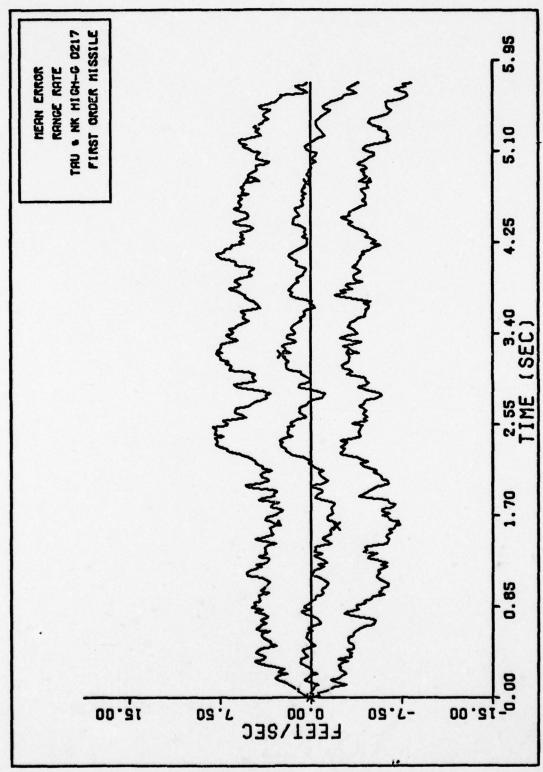


Fig. 237. RANGE RATE FIRST ORDER MISSILE

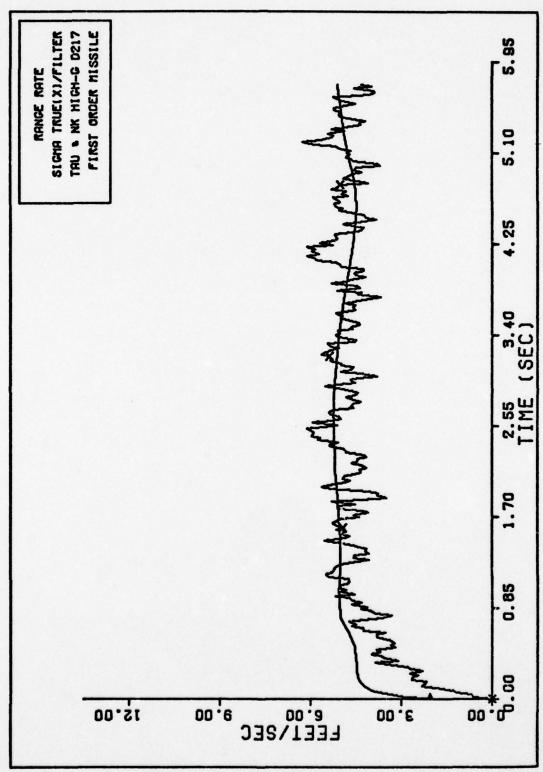


Fig. 238. RANGE RATE SIGMAS FIRST ORDER

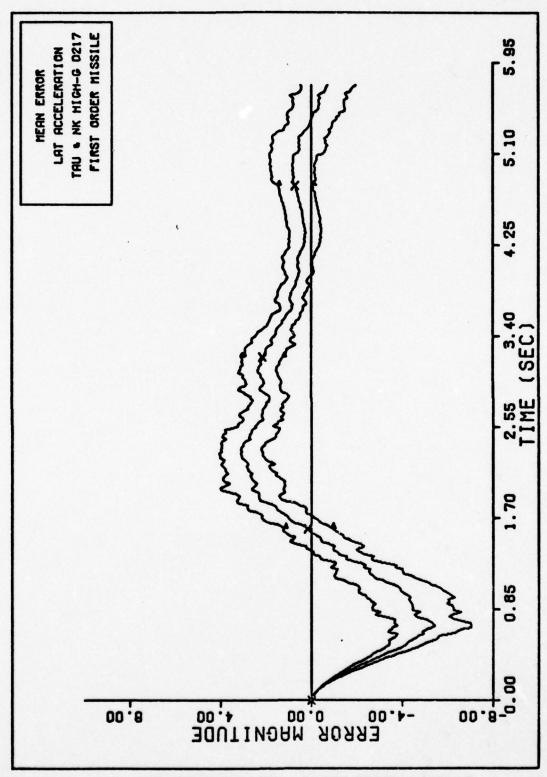


Fig. 239. LAT ACCELERATION FIRST ORDER MISSILE

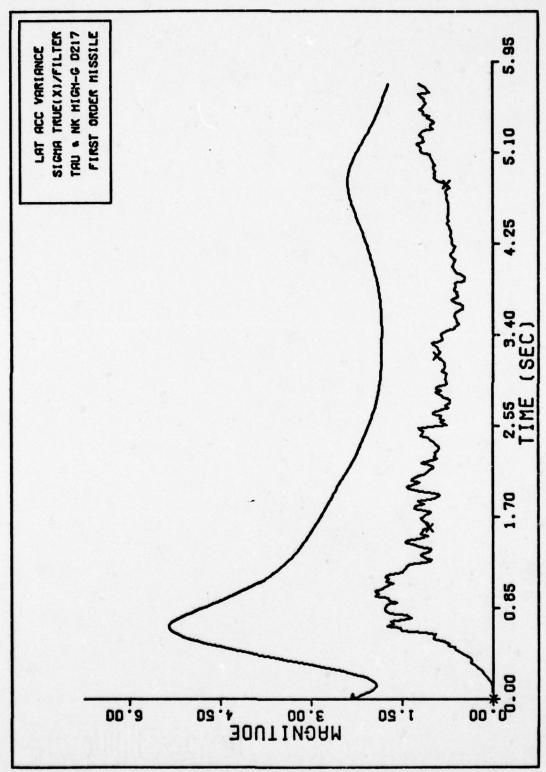


Fig. 240. LAT ACCELERATION SIGMAS FIRST ORDER

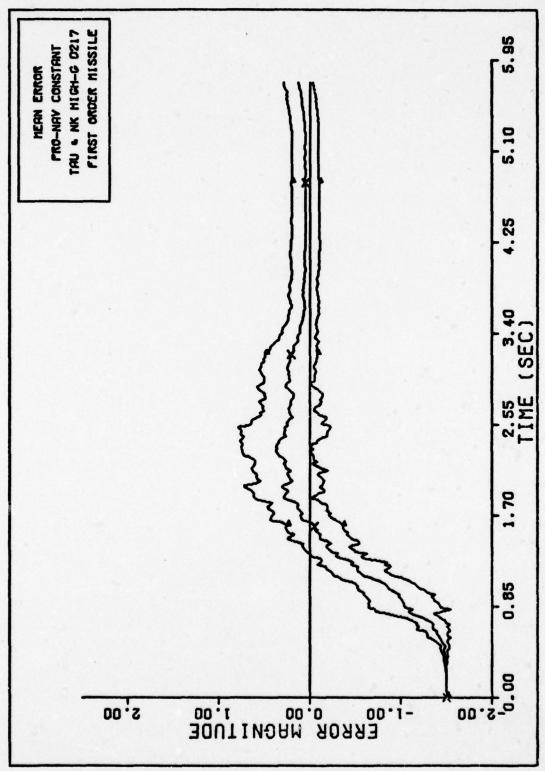


Fig. 241. PRO-NAV CONSTANT FIRST ORDER MISSILE

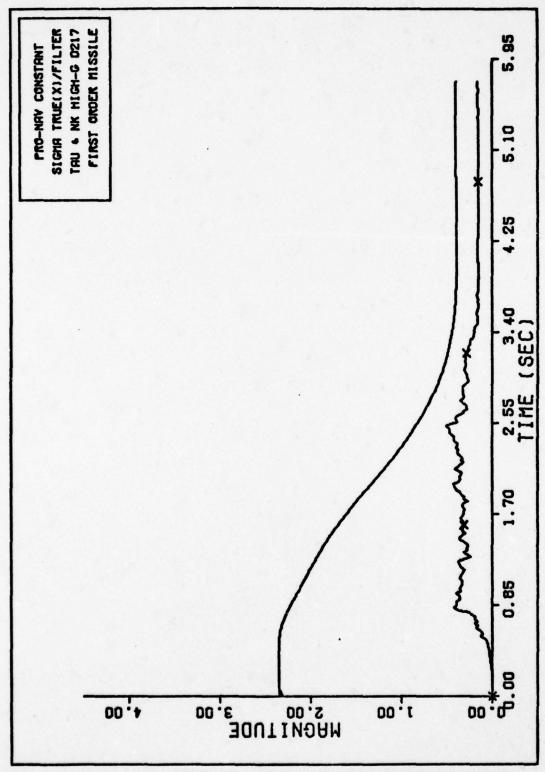


Fig. 242. PRD-NAV CONSTANT SIGMAS FIRST ORDER

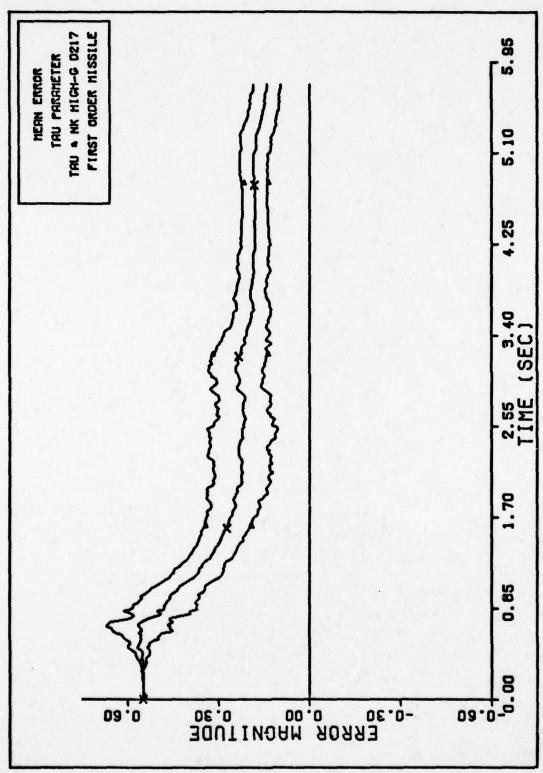


Fig. 243. TAU PARAMETER FIRST ORDER MISSILE

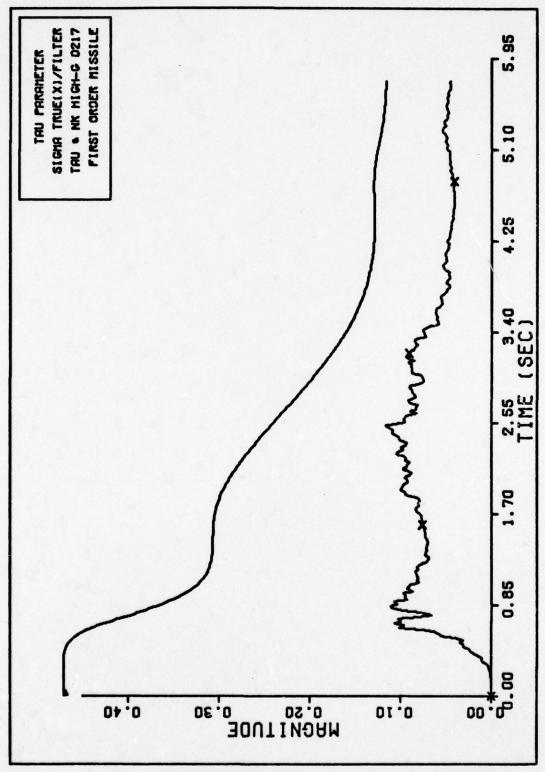


Fig. 244. TAU PARAMETER SIGMAS FIRST ORDER

## n and Tf Estimation (low-g scenario)

The initial state estimates and the tuning parameters for this case are

$$V_{mx}^{I}(0) = 1225.7 \text{ fps}$$

$$V_{mx}^{I}(0) = 1225.7 \text{ fps}$$
  
 $\dot{\theta}(0) = 4.363345 \text{ radians}$   
 $R(0) = 10000. \text{ feet}$ 

$$R(0) = 10000.$$
 feet

$$\dot{R}(0) = -2122. \text{ fps}$$

$$a_{T_i}(0) = 0.$$

$$n(0) = 3.$$

$$\tau_{\mathbf{f}}(0) = 1.5 \text{ seconds}$$

$$M/S(0) = 29.197 slugs/ft^2$$

$$\underline{\mathbf{R}} = \begin{bmatrix} 3.E-5 & 0. & 0. \\ 0. & 500. & 0. \\ 0. & 0. & 100. \end{bmatrix}$$

These plots were generated as a test case to determine if the parameters could be estimated for a low-g scenario. Only n and  $\tau_f$  were estimated along with the five dynamic states of the missile model. These plots can be compared with Figures 231 through 244 which were generated using the high-g scenario. The n and  $\tau_f$  in the filter, for this set, were initialized at 3.0 and 1.5 seconds respectively. The true n was 4.5 and thr true value for  $\tau_f$  was defined as 0.85 seconds.

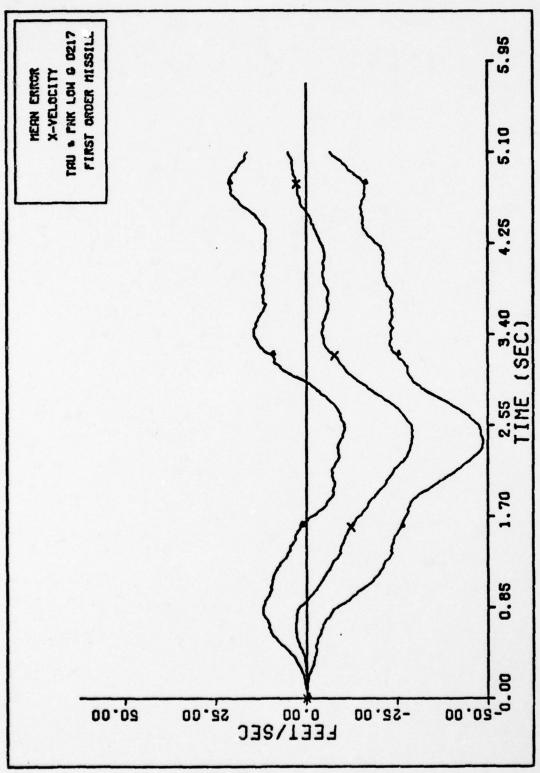


Fig. 245. X-VELOCITY FIRST ORDER MISSILE

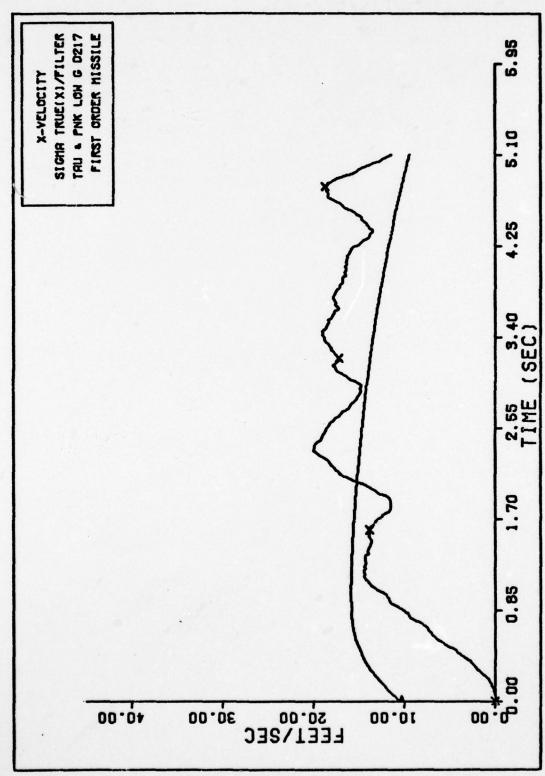


Fig. 246. X-VELOCITY SIGMAS FIRST DRDER

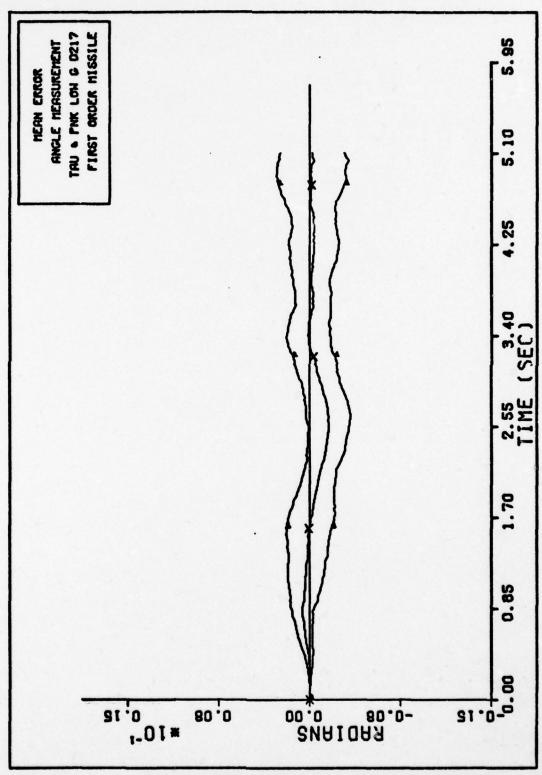


Fig. 247. ANGLE MEASUREMENT FIRST ORDER MISSILE

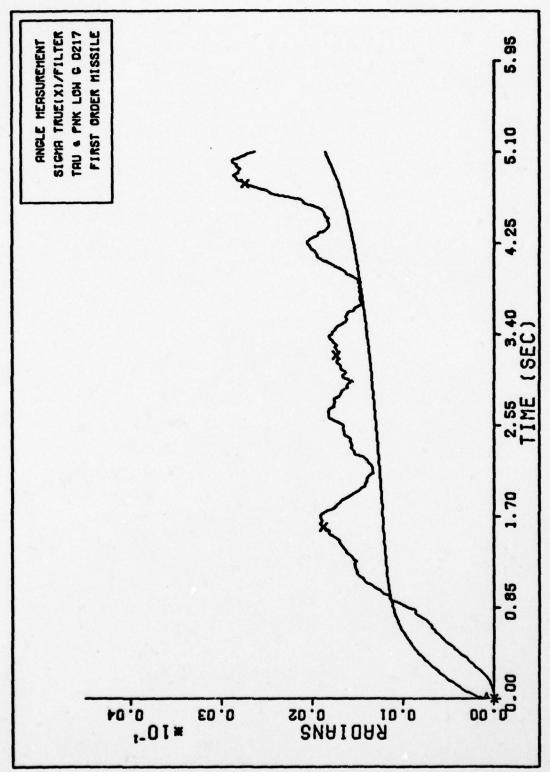


Fig. 248. ANGLE MEASUREMENT SIGMAS FIRST ORDER

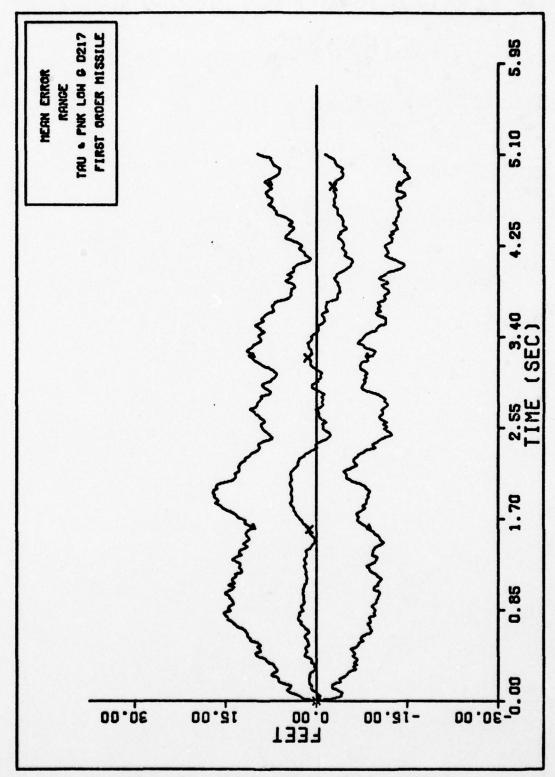


Fig. 249. RANGE FIRST ORDER MISSILE

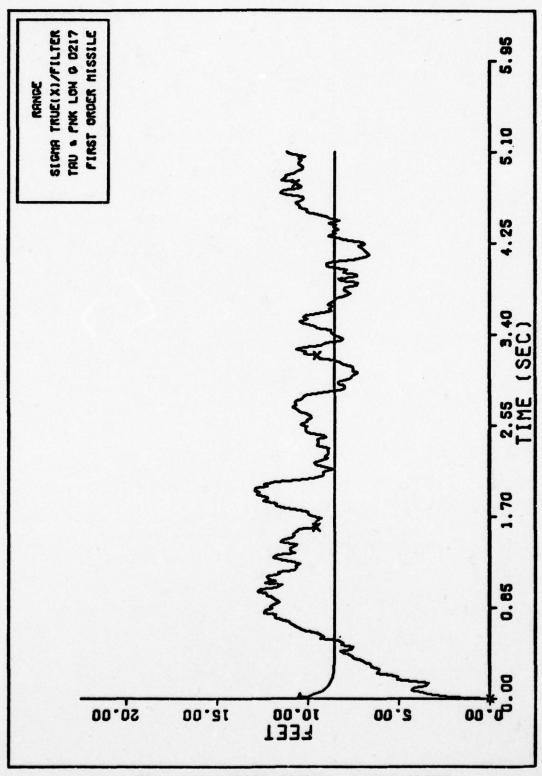


Fig. 250. RANGE SIGMAS FIRST ORDER

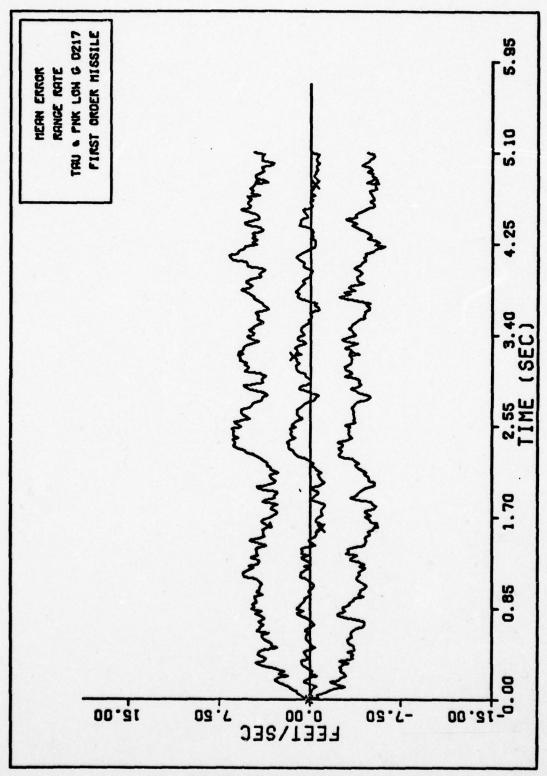


Fig. 251. RANGE RATE FIRST ORDER MISSILE

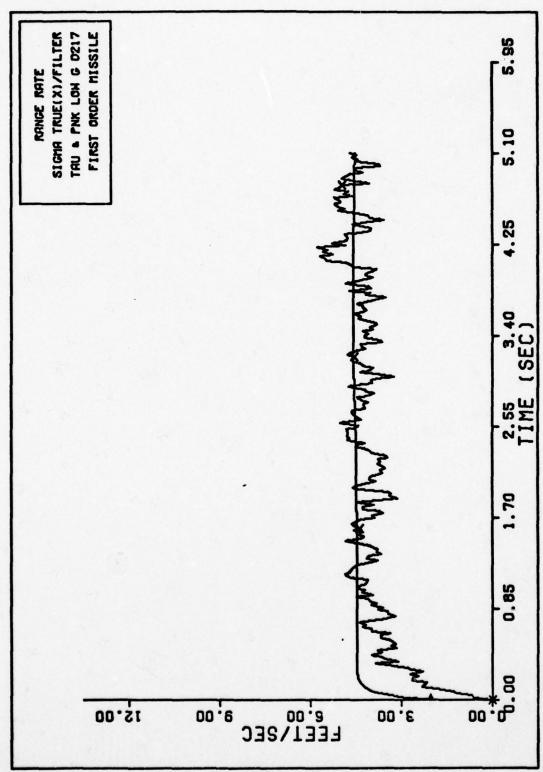


Fig. 252. RANGE RATE SIGMAS FIRST ORDER

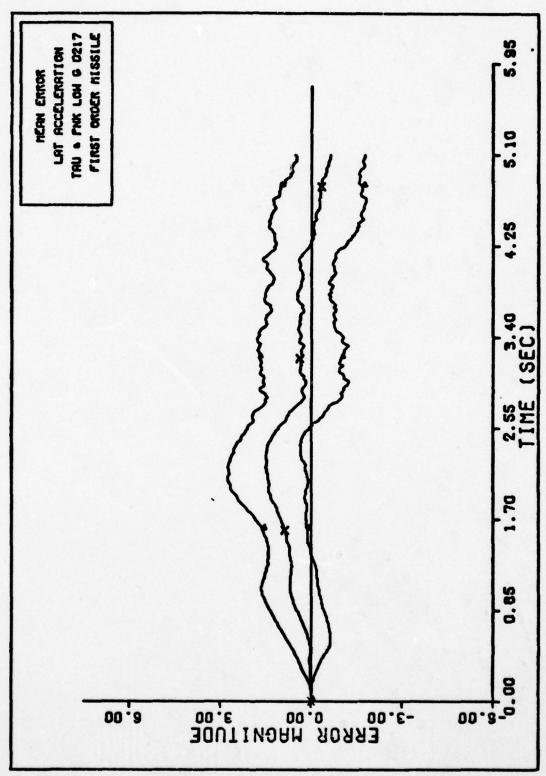


Fig. 253. LAT ACCELERATION FIRST ORDER MISSILE

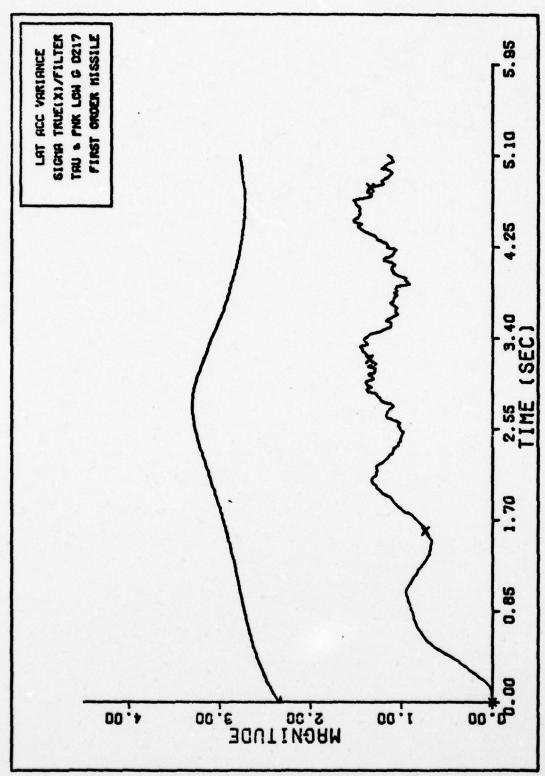


Fig. 254. LAT ACCELERATION SIGMAS FIRST ORDER

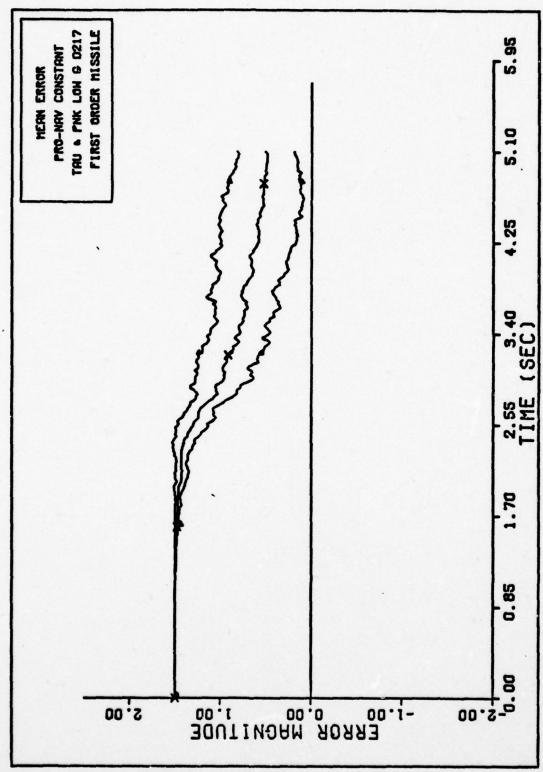


Fig. 255. PRO-NAY CONSTANT FIRST ORDER MISSILE

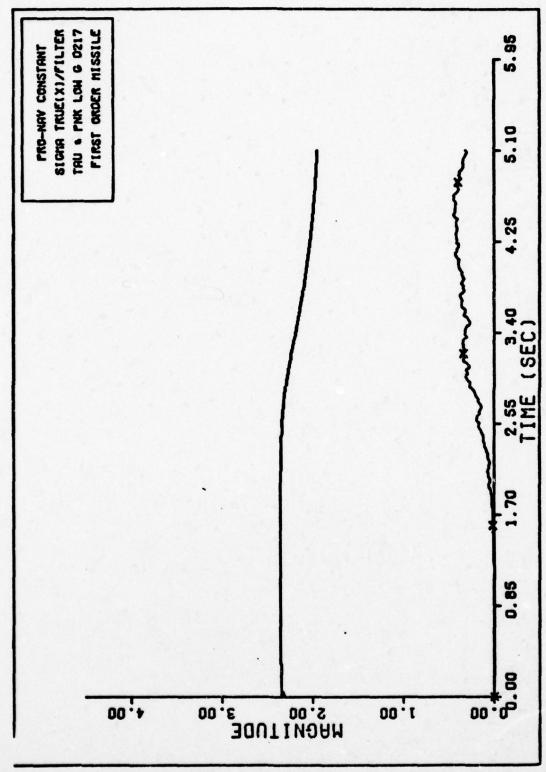


Fig. 256. -RO-NAV CONSTANT SIGMAS FIRST ORDER

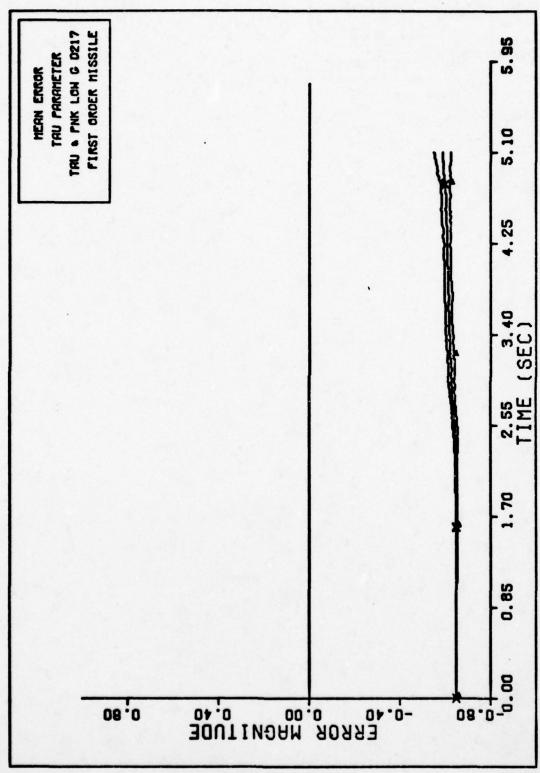


Fig. 257. TAU PARAMETER FIRST ORDER MISSILE

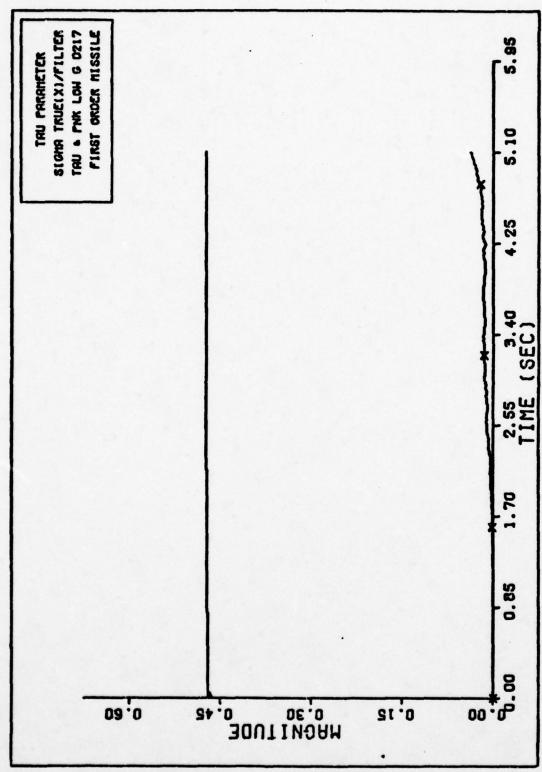


Fig. 258. TAU PARAMETER SIGMAS FIRST ORDER

## n, Tf, and M/S Estimation

The initial state estimates and the tuning parameters for this case are

$$v_{mx}^{I}(0) = 1225.7 \text{ fps}$$
 $\dot{v}_{mx}(0) = 4.363345 \text{ radians}$ 
 $R(0) = 10000. \text{ feet}$ 
 $\dot{R}(0) = -2122. \text{ fps}$ 
 $a_{L}(0) = 0.$ 
 $n(0) = 6.$ 
 $\tau_{f}(0) = .3 \text{ seconds}$ 
 $M/S(0) = 15. \text{ slugs/ft}^{2}$ 

$$\underline{\mathbf{R}} = \begin{bmatrix} \mathbf{\overline{2.E-5}} & 0. & 0. \\ 0. & 500. & 0. \\ 0. & 0. & 100. \end{bmatrix}$$

These plots were generated to demonstrate the first order filter's ability to estimate all three parameters and all five dynamic states simultaneously. The states were not initialized with any error. The parameter were initialized in the filter as follows

$$n(0) = 6.0 , true value = 4.5$$
 
$$\tau_{\bf f}(0) = 0.3 \ {\rm seconds} , true value = .85 \ {\rm seconds}$$
 
$$M/S(0) = 15. \ {\rm slugs/ft}^2 , true \ value = 29.197 \ {\rm slugs/ft}^2$$

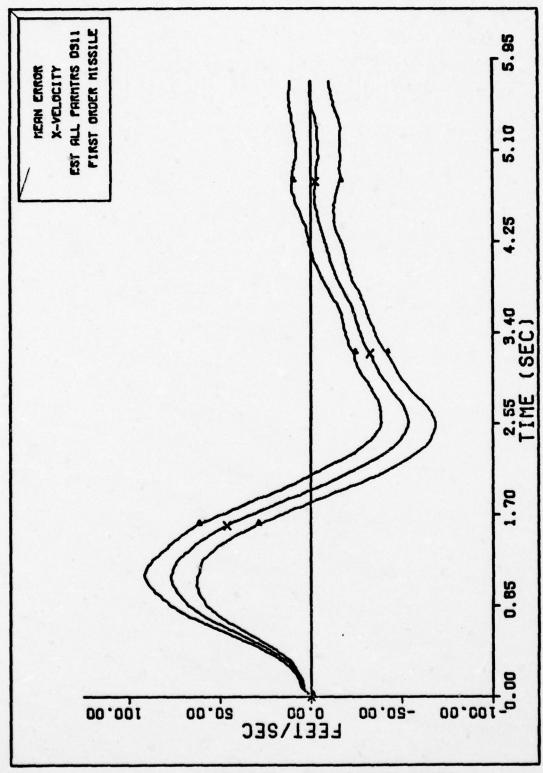


Fig. 259. X-VELOCITY FIRST ORDER MISSILE

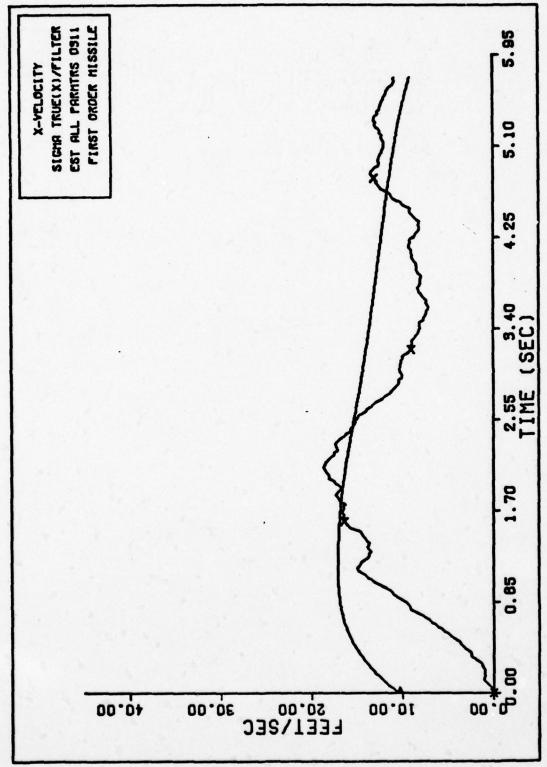


Fig. 260. X-VELOCITY SIDMAS FIRST DRDER

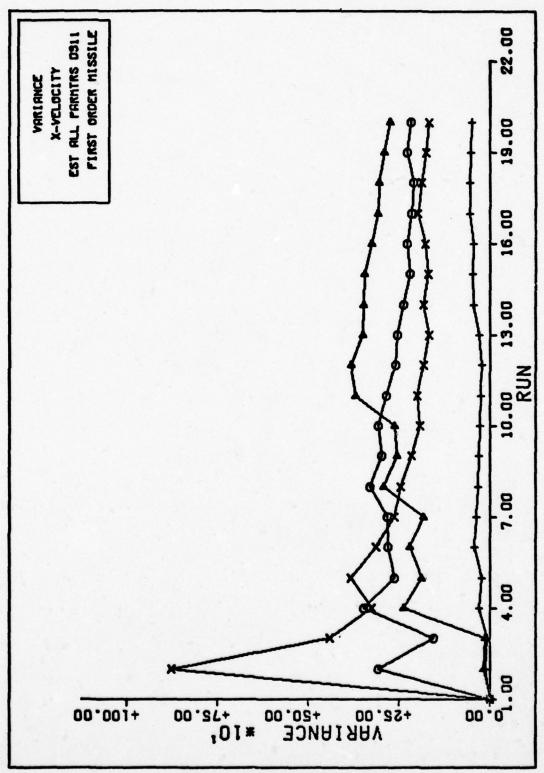


Fig. 261. VARIANCE CONVERGENCE

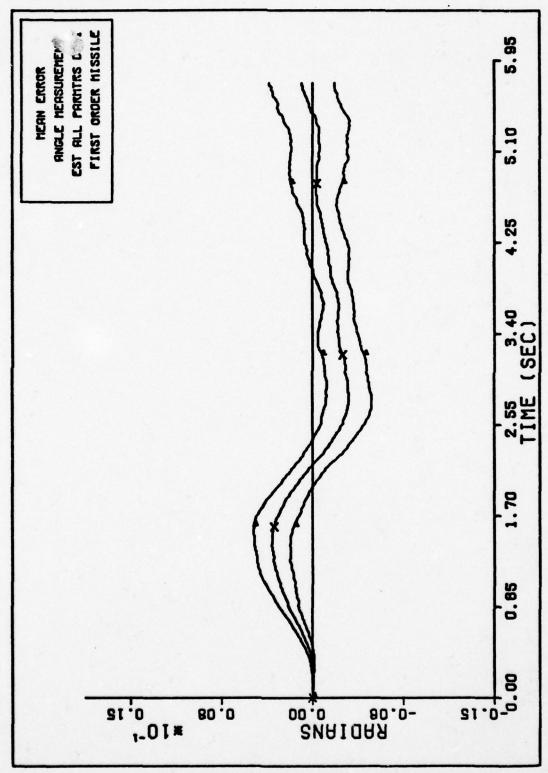


Fig. 262. ANGLE MEASUREMENT FIRST ORDER MISSILE

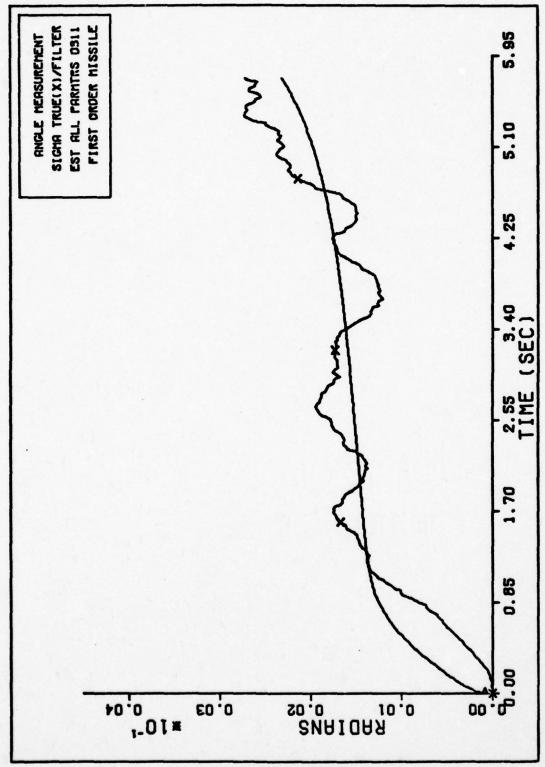


Fig. 263. ANGLE MEASUREMENT SIGMAS FIRST ORDER

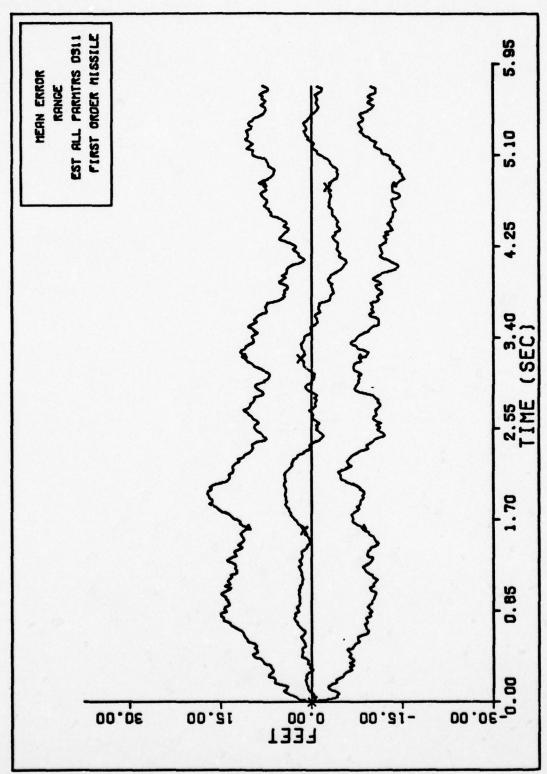


Fig. 264. RANGE FIRST ORDER MISSILE

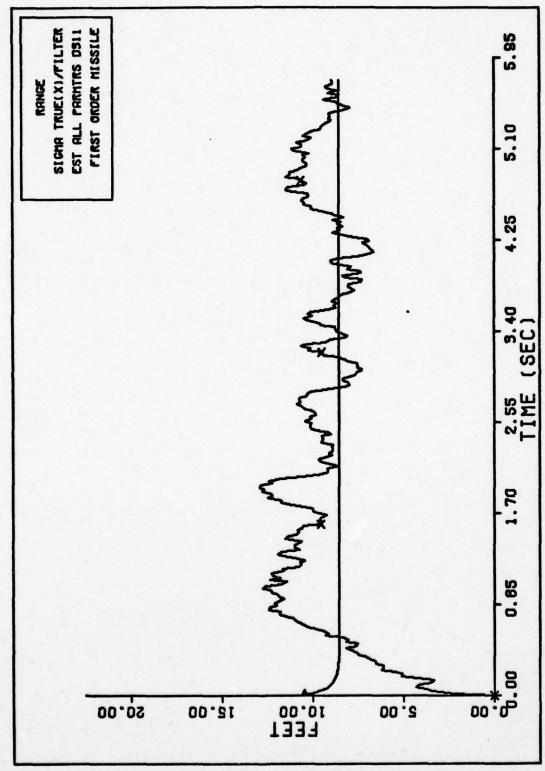


Fig. 265. RANGE SIGMAS FIRST ORDER

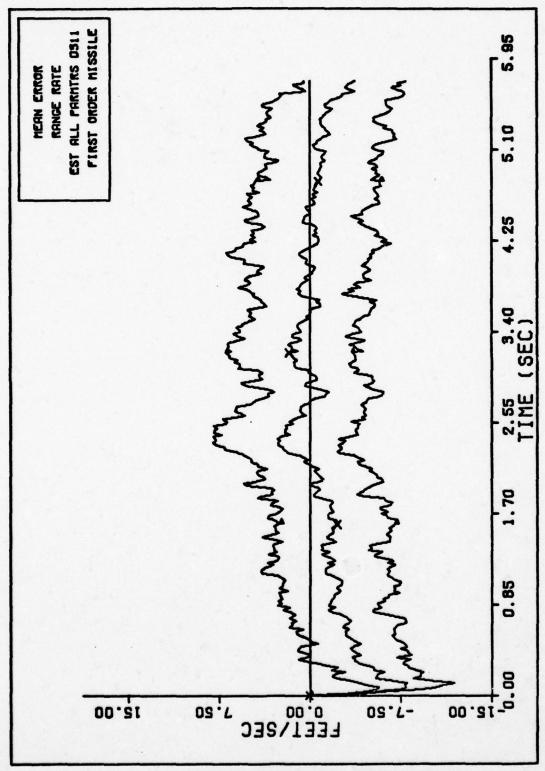


Fig. 266. RANGE RATE FIRST ORDER MISSILE

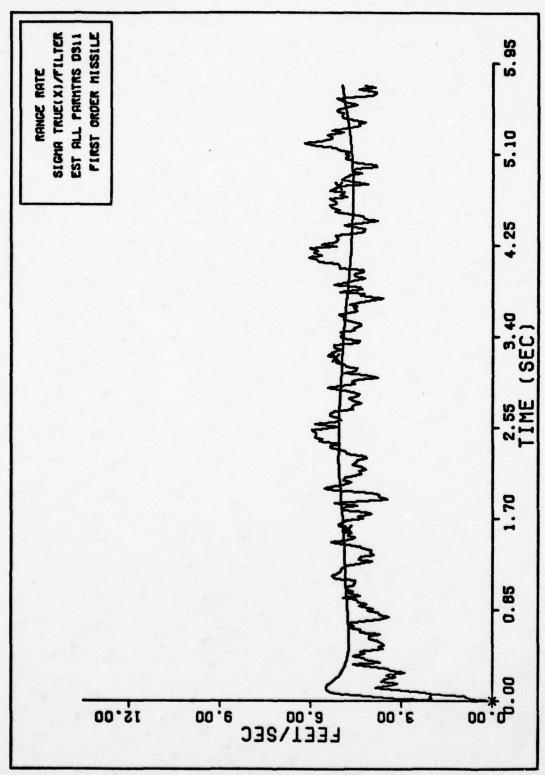


Fig. 267. RANGE RATE SIGMAS FIRST ORDER

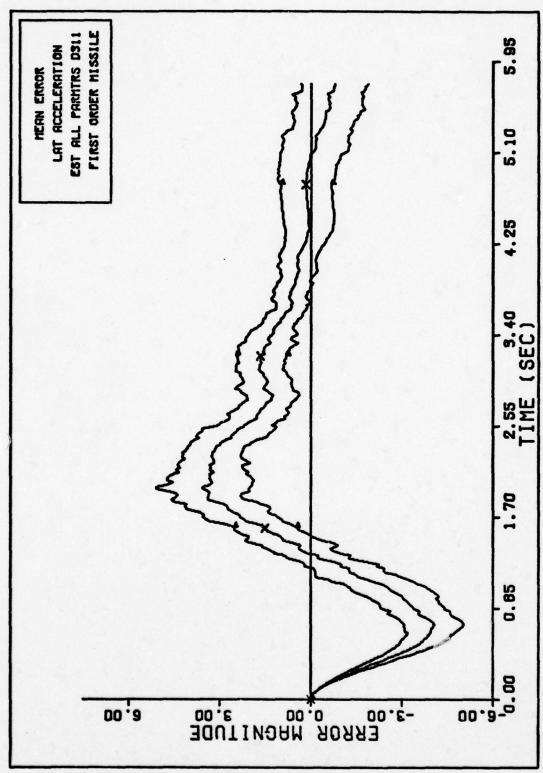


Fig. 268. LAT ACCELERATION FIRST ORDER MISSILE

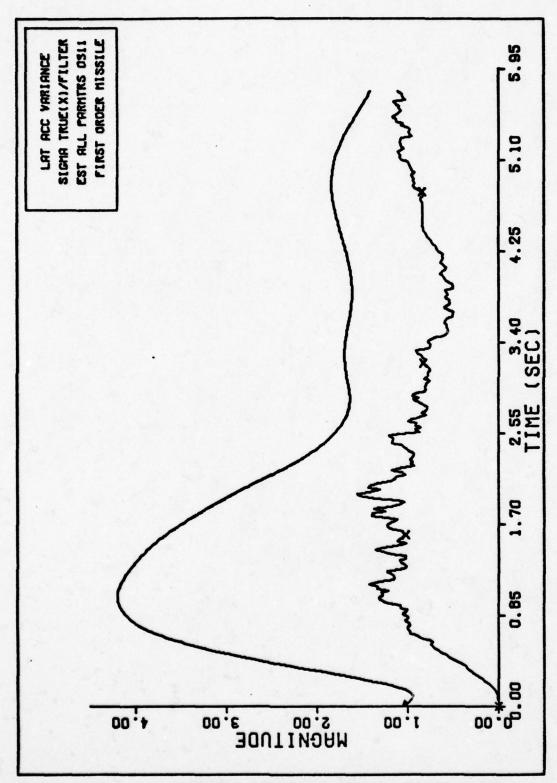


Fig. 269. LAT ACCELERATION SIGMAS FIRST ORDER

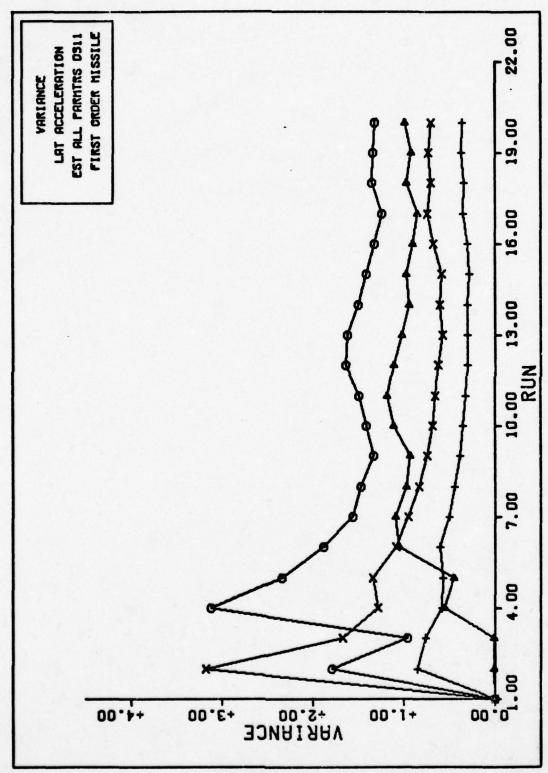


Fig. 270. VARIANCE CONVERGENCE

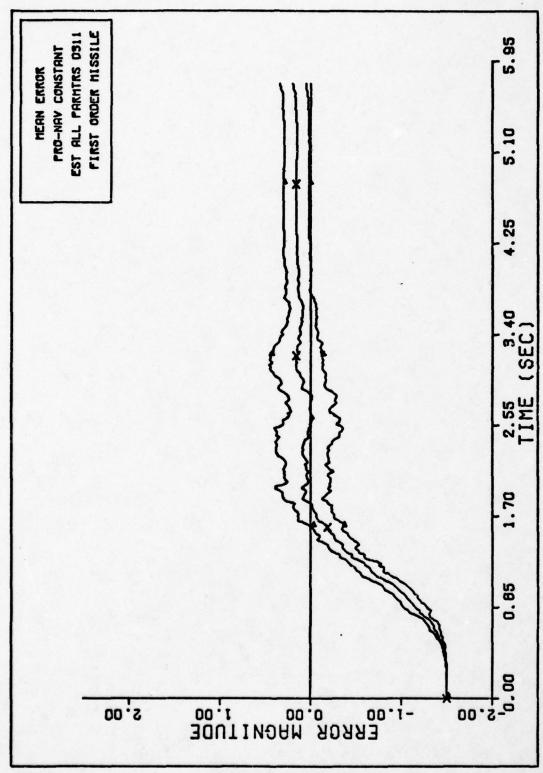


Fig. 271. PRO-NAV CONSTANT FIRST ORDER MISSILE

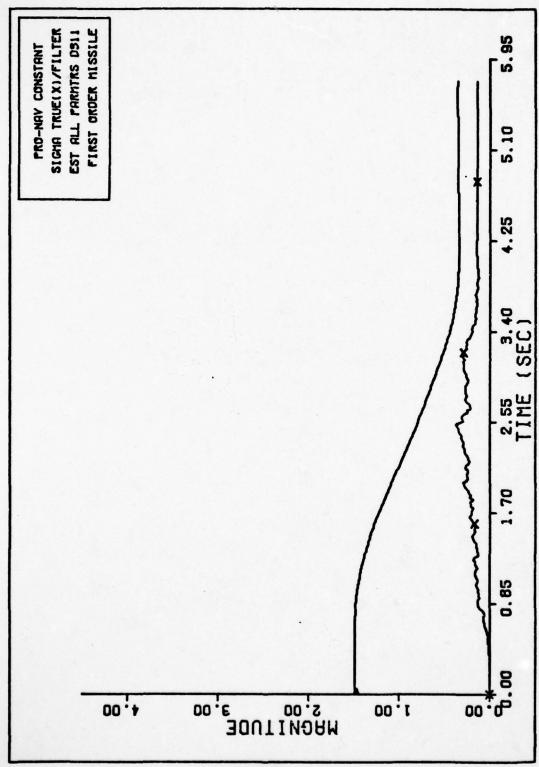


Fig. 272. PRO-NAV CONSTANT SIGMAS FIRST ORDER

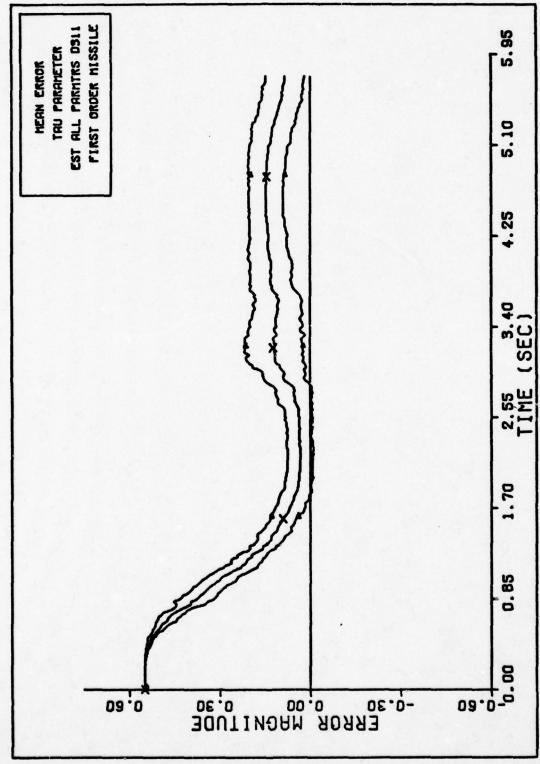


Fig. 273. TAU PARAMETER FIRST ORDER MISSILE

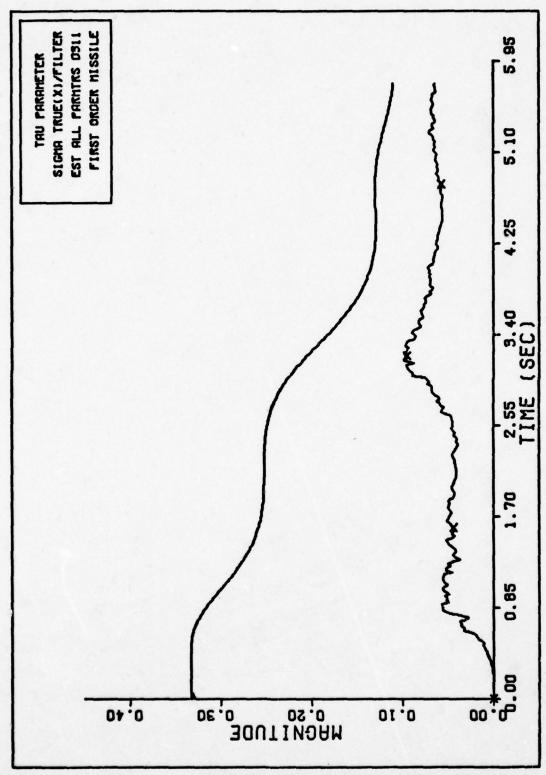


Fig. 274. TAU PARAMETER SIGMAS FIRST ORDER

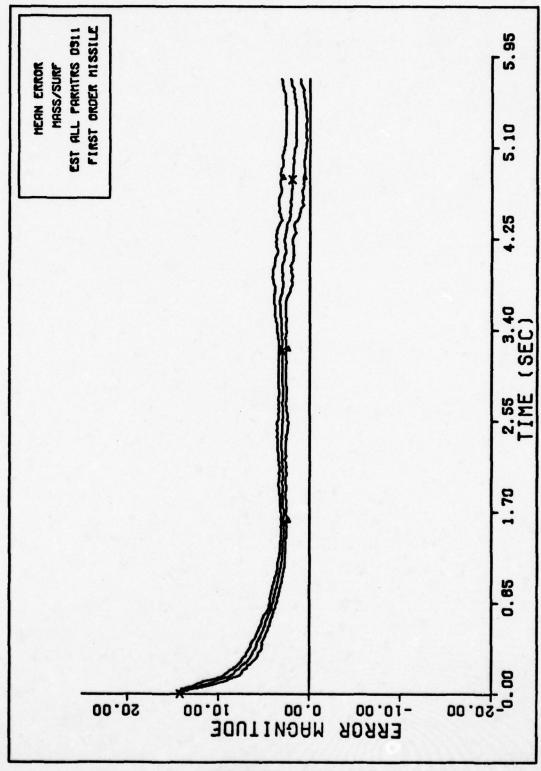


Fig. 275. MASS/SURF FIRST DRDER MISSILE

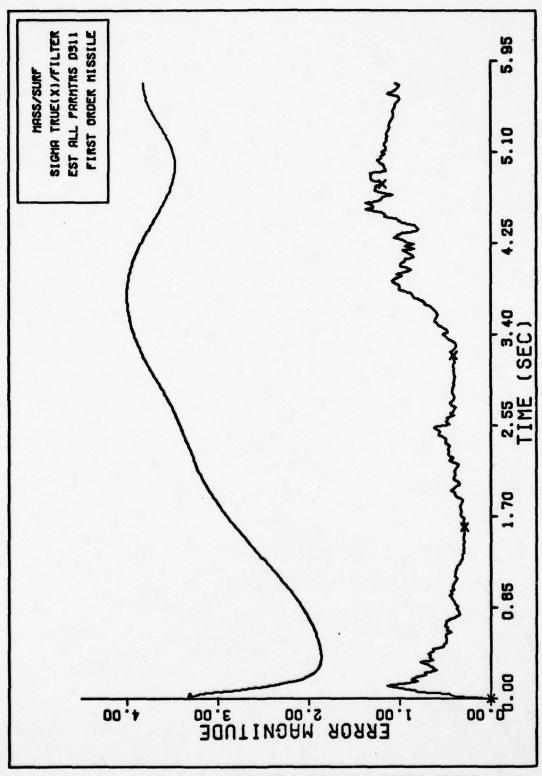


Fig. 276. MASS/SURF SIGMAS FIRST ORDER

## n, If. and M/S Estimation with Initial State Errors

The initial state estimates and the tuning parameters for this case are

$$V_{mx}^{I}(0) = 1000$$
. fps  
 $\dot{\theta}(0) = 4.343345$  radians  
 $R(0) = 9000$ . feet  
 $\dot{R}(0) = -1900$ . fps  
 $a_{L}(0) = 15$ . g's  
 $n(0) = 6$ .  
 $T_{f}(0) = .3$  seconds

$$M/S(0) = 15$$
. slugs/ft<sup>2</sup>

$$\underline{\mathbf{R}} = \begin{bmatrix} \overline{\mathbf{3}}.\mathbf{E}-5 & 0. & 0. \\ 0. & 500. & 0. \\ 0. & 0. & 100. \end{bmatrix}$$

This was the final evaluation of the first order filter. The states and parameters were both initialized with large errors. The initial values of the filter's states are given on the previous page. The true values for each are listed below.

$$V_{mx}^{I}(0) = 1225.6 \text{ fps}$$
  
 $\dot{\theta}(0) = 4.363345 \text{ radians}$   
 $R(0) = 10000. \text{ feet}$   
 $\dot{R}(0) = -2122. \text{ fps}$   
 $a_{L}(0) = 0.$   
 $n(0) = 4.5$   
 $\tau_{f}(0) = .85 \text{ seconds}$   
 $M/S(0) = 29.197 \text{ slugs/ft}^2$ 

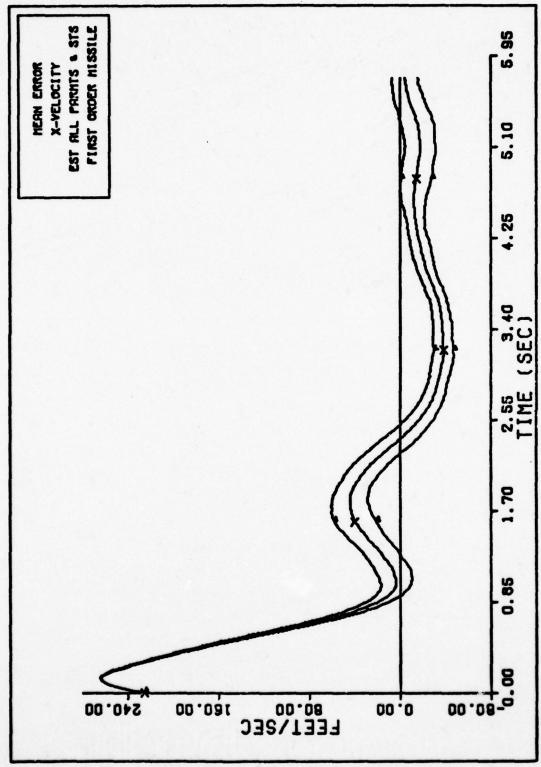


Fig. 277. X-VELOCITY FIRST ORDER MISSILE

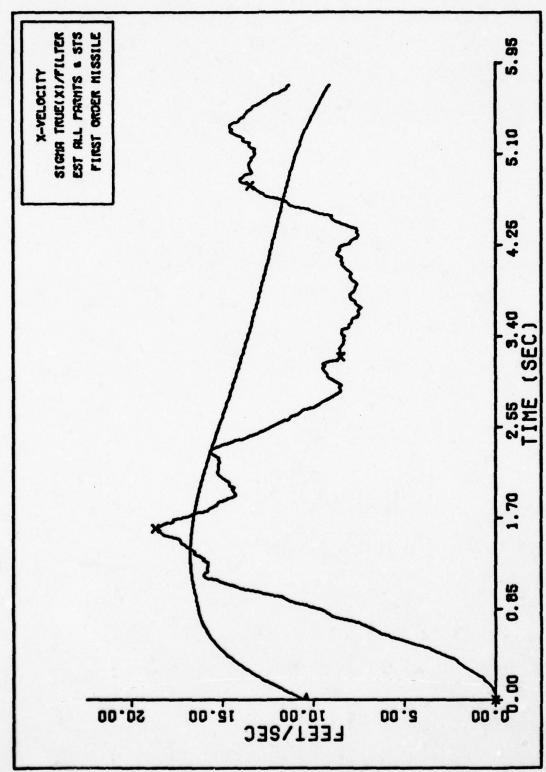


Fig. 278. X-VELOCITY SIGMAS FIRST ORDER

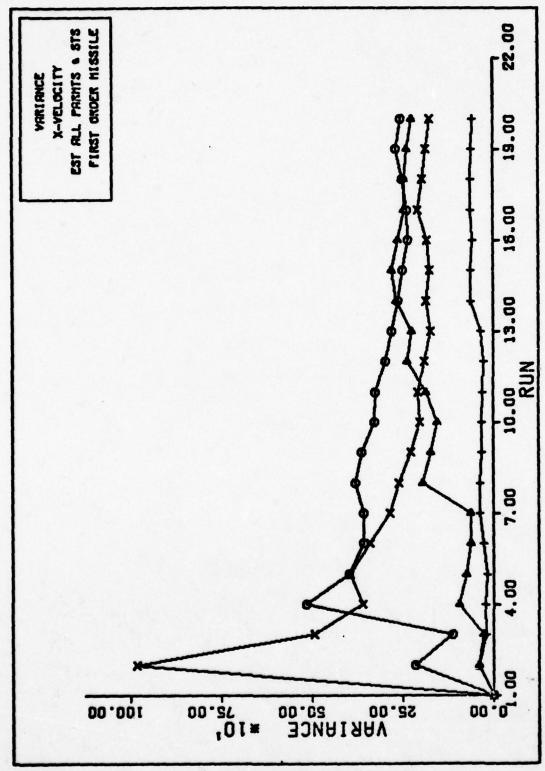


Fig. 279. VARIANCE CONVERGENCE

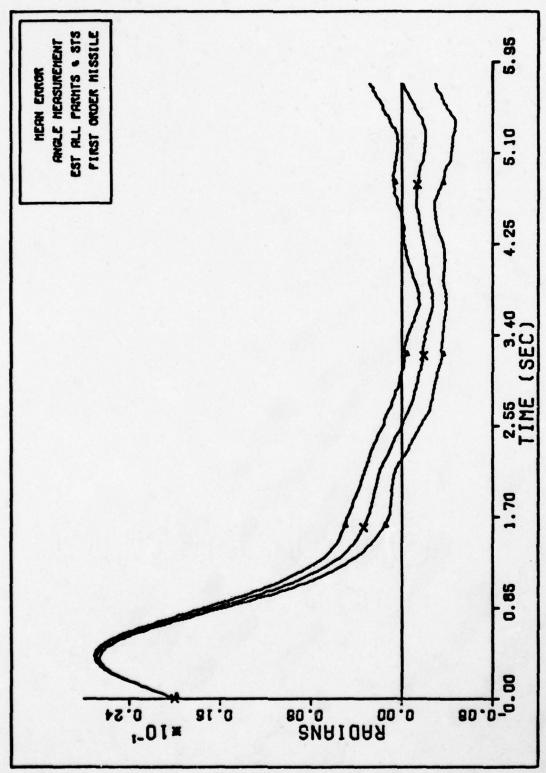


Fig. 280. ANGLE MERSUREMENT FIRST ORDER MISSILE

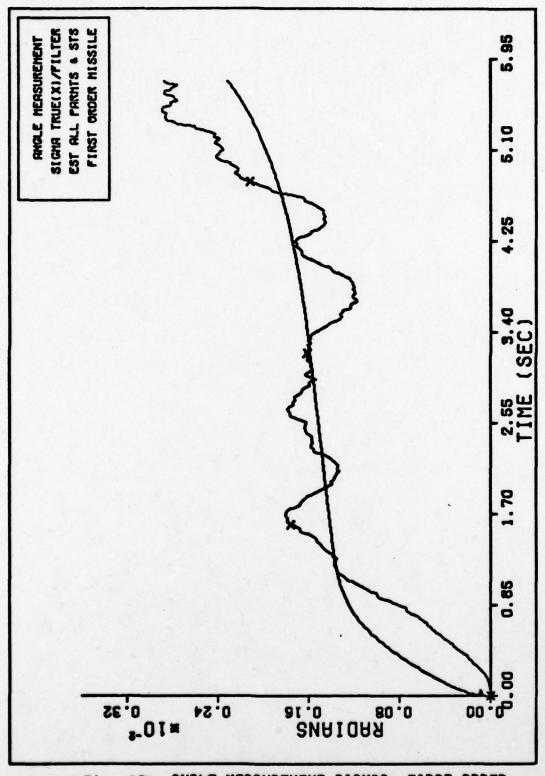


Fig. 281. ANGLE MERSUREMENT SIGMAS FIRST DRDER

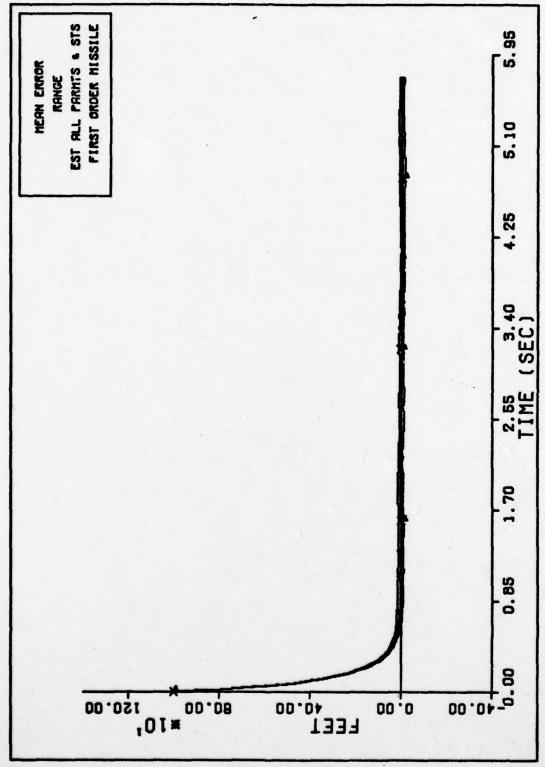


Fig. 282. RANGE FIRST DRDER MISSILE

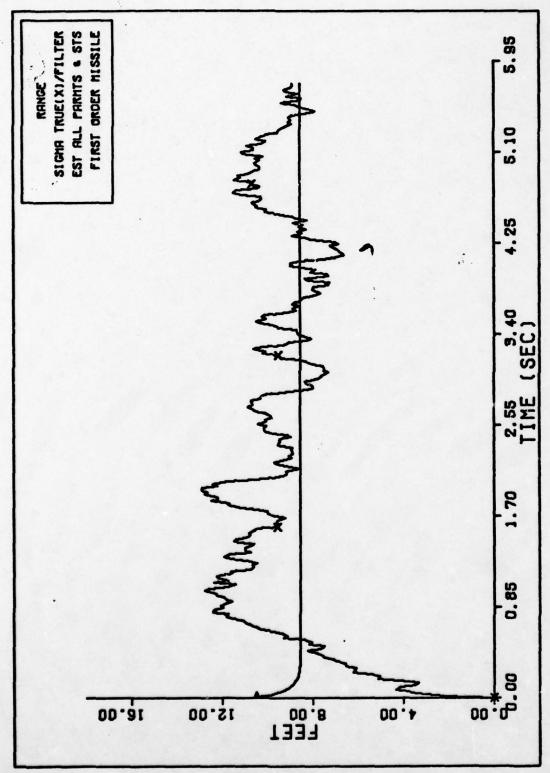


Fig. 283. RANGE SIGMAS FIRST ORDER

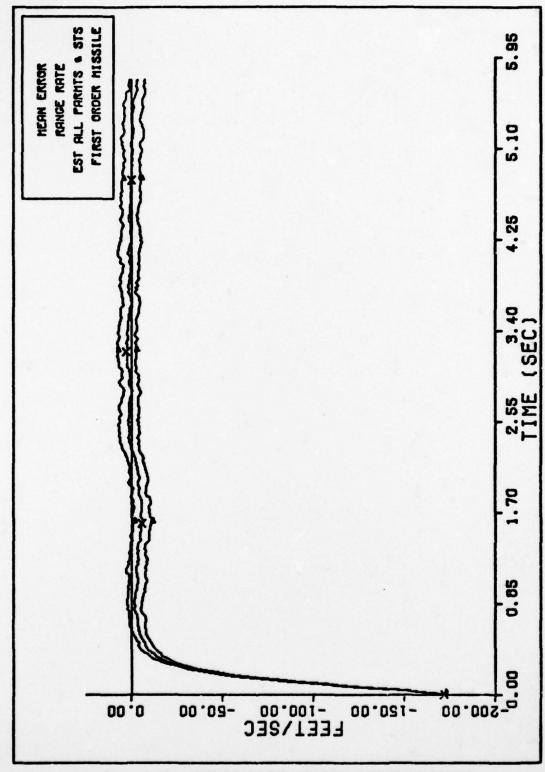


Fig. 284. RANGE RATE FIRST ORDER MISSILE

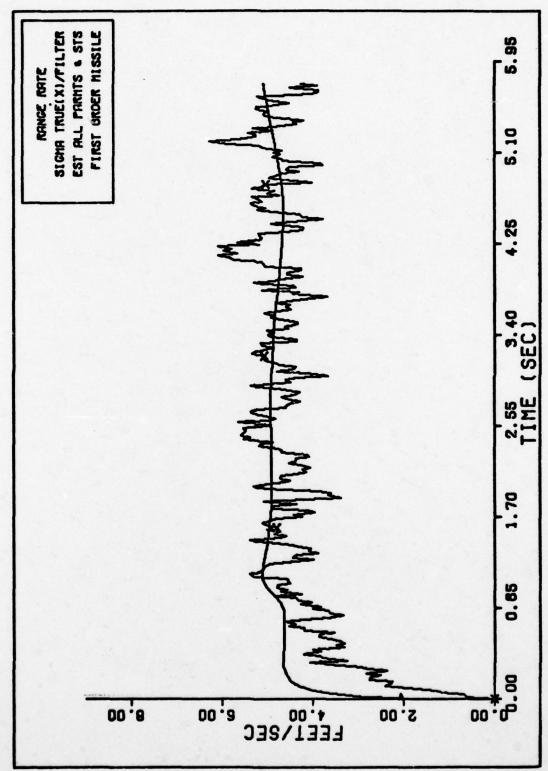


Fig. 285. RANGE RATE SIGNAS FIRST ORDER

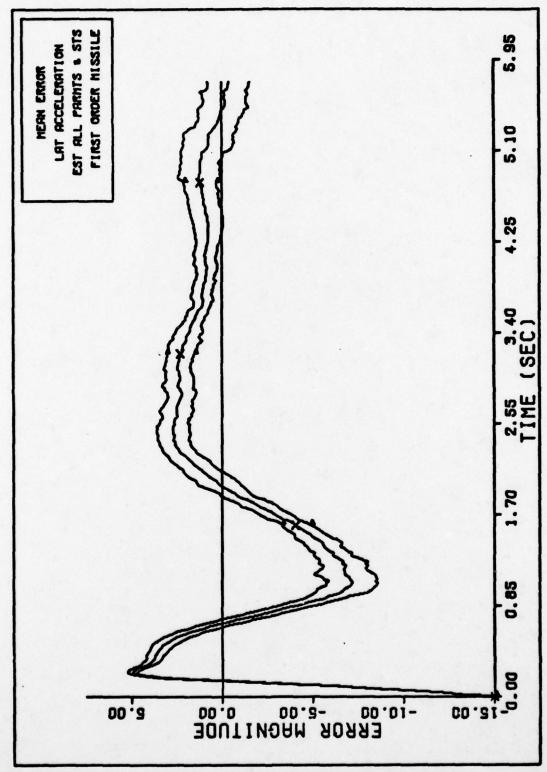


Fig. 286. LAT ACCELERATION FIRST ORDER MISSILE

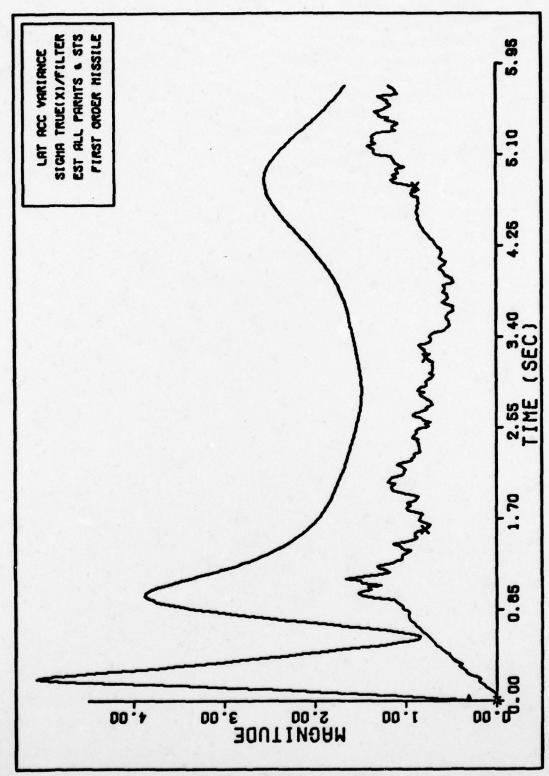


Fig. 287. LAT ACCELERATION SIGMAS FIRST ORDER

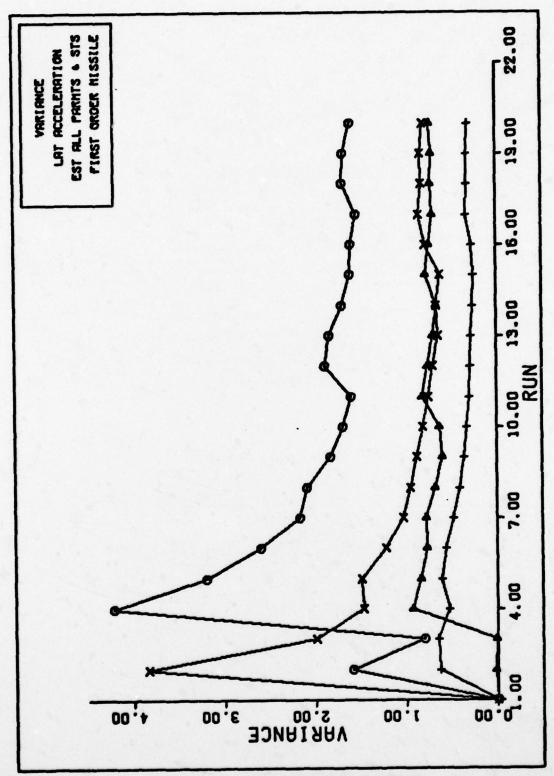


Fig. 288. VARIANCE CONVERGENCE

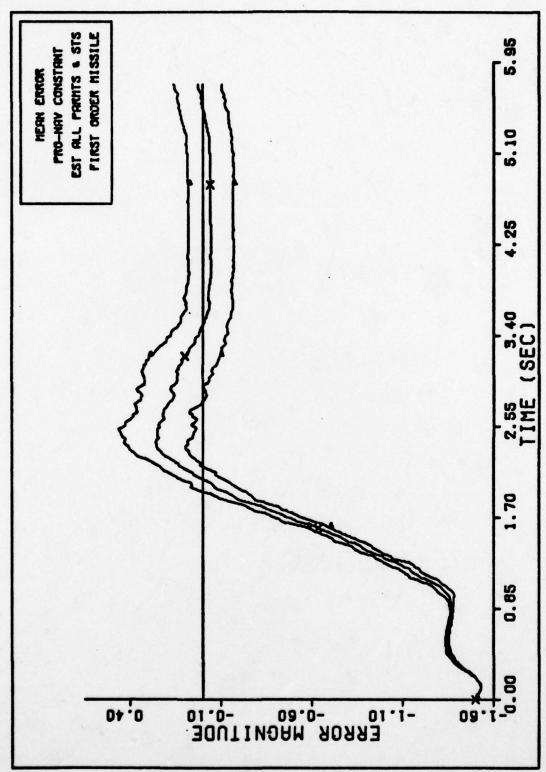


Fig. 289. PRO-NAV CONSTANT FIRST ORDER MISSILE

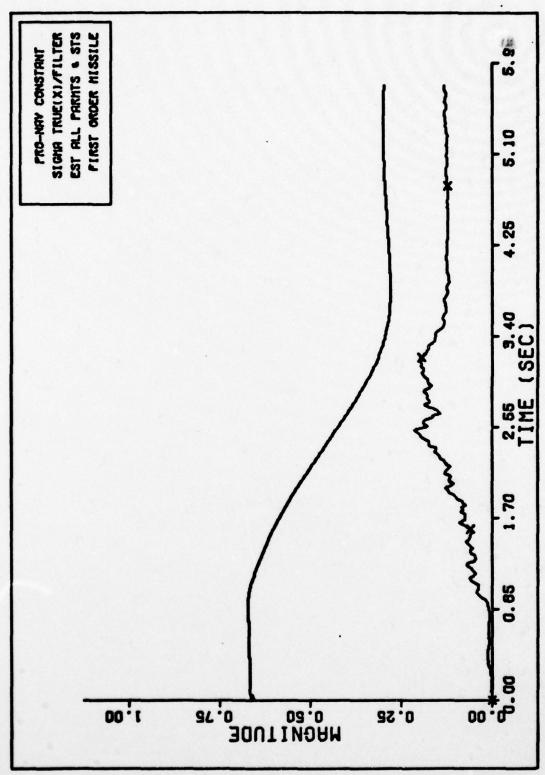


Fig. 290. PRO-NAV CONSTANT SIGMAS FIRST ORDER

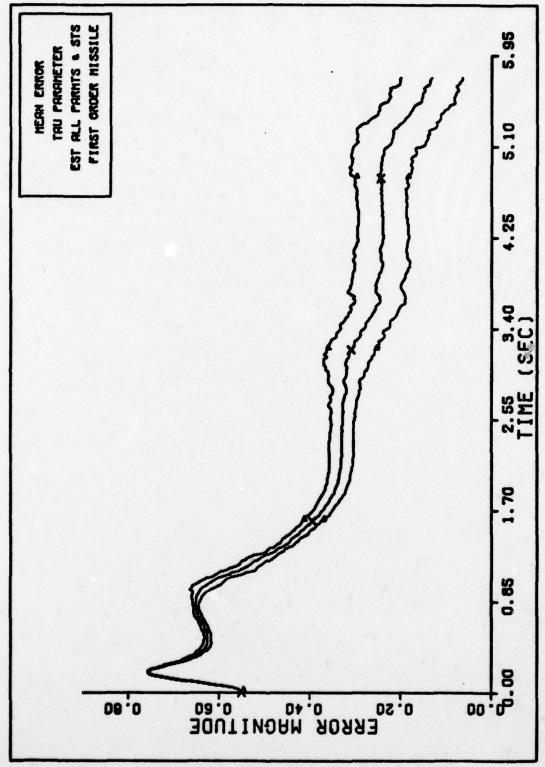


Fig. 291. TAU PARAMETER FIRST ORDER MISSILE

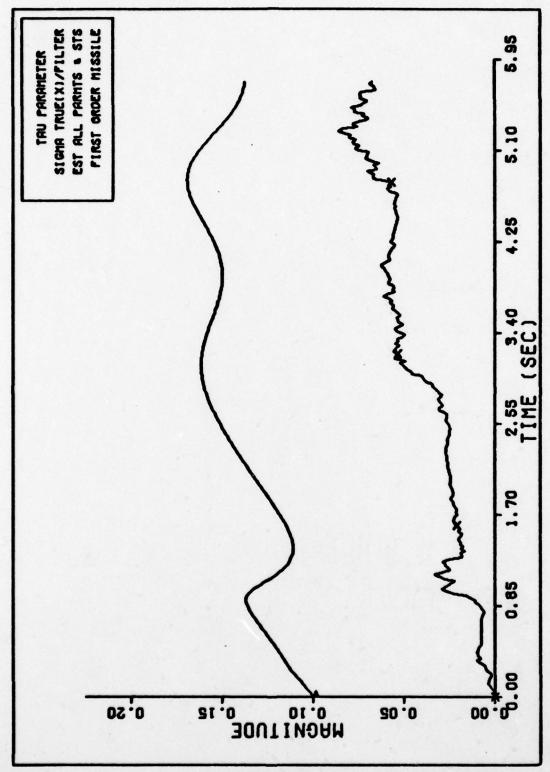


Fig. 292. TAU PARAMETER SIGMAS FIRST ORDER

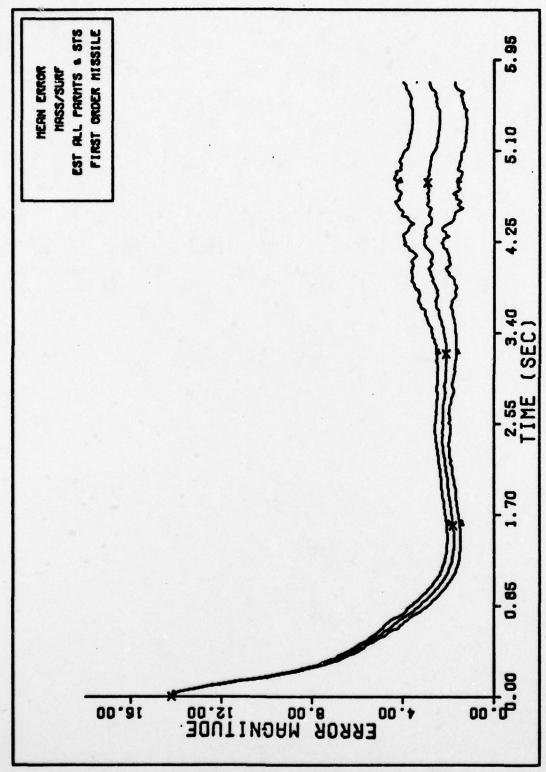


Fig. 293. MASS/SURF FIRST ORDER MISSILE

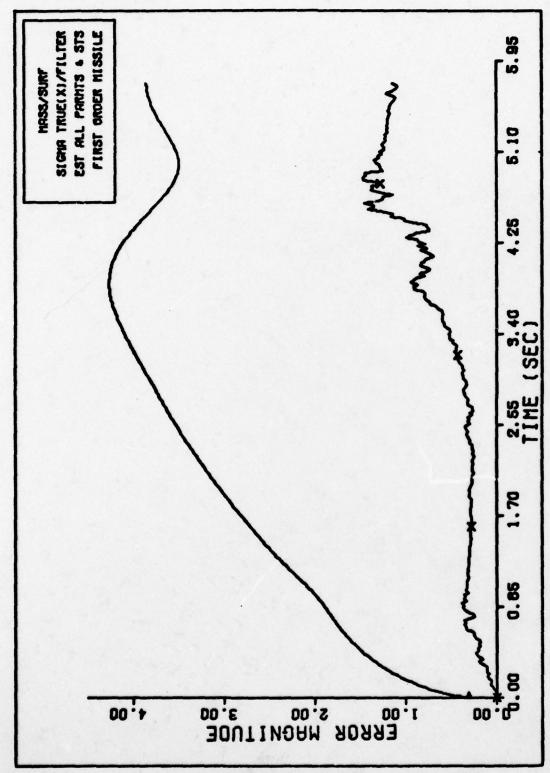


Fig. 294. MASS/SURF SIGMAS FIRST ORDER